Louis Savinien Dupuis Journal of Multidisciplinary Research, 2025: V(4) 65–68
DOI: 10.21839/lsdjmr.2025.v4.248

Research Article

PERFORMANCE MODELING OF M/M/1 QUEUES WITH K-CONSECUTIVE VACATIONS AND CUSTOMER RENEGING

V. Vedhadharshini *

Department of Mathematics, Immaculate College for Women, Cuddalore, India

(Received: 09-04-2025; Accepted 08-06-2025; Published Online 03-11-2025)

Abstract

This paper investigates an M/M/1 queueing system incorporating variant working vacations(VWV) and customer reneging, which reflects realistic service environments where servers operate at reduced efficiency during vacation periods and customers may leave due to impatience. The system is modelled as a continuous-time Markov chain (CTMC) with states defined by customer count and server status. The matrix geometric method is employed to derive steady-state probability distributions. Using these results, key performance measures such as expected queue length, server utilization, and probability of system states are evaluated. Numerical analysis demonstrates the impact of system parameters, including arrival rate, service rate, vacation rate, and reneging rate. The results show that increasing service rate improves system efficiency, while higher reneging and vacation rates significantly influence queue dynamics. This model provides useful insights for optimizing real-world service systems.

.

Keywords: *M/M/1 queue, variant working vacation, customer reneging, matrix geometric method, Markov process, steady-state analysis*

Introduction

Queueing systems are fundamental in modelling real-life waiting situations such as communication networks, manufacturing systems, and service industries. Traditional queueing models assume continuous service; however, in many practical situations, servers may take vacations or operate at reduced capacity.

The variant working vacation (VWV) policy allows the server to take up to *K* consecutive working vacations, during which service continues at a reduced rate. Additionally, customer impatience is modelled through reneging, where customers may leave the system before receiving service.

This study extends classical queueing models by integrating:

Multiple working vacations

Limited vacation policy (K-consecutive vacations)

Customer reneging

The matrix geometric method is used to analyse the system and derive performance measures.

Model Description

We consider an M/M/1 queue with the following assumptions:

Arrival process: Poisson with rate λ

Service time: Exponential with rate μ

Vacation service rate: η, where $\eta < \mu$

Vacation time: Exponentially distributed with rate θ

Maximum vacations: K

Reneging rate: α

System Behaviour

When the system becomes empty, the server enters a working vacation.

If no arrivals occur, vacations continue up to K times.

After K vacations, the server returns to normal service.

Customers may leave due to impatience (reneging).

The system state is defined as:$(L(t), J(t))$Type equation here.

where:

$L(t)$: number of customers

$J(t)$: server state

The Infinitesimal Generator Matrix Q is a "block-tridiagonal" matrix:

$$Q=\begin{pmatrix} A_0 & C & 0 & \cdots \\ B_1 & A_1 & C & \cdots \\ 0 & B_2 & A_2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

And the elements of the matrix Q are given by

$$C = \begin{pmatrix} \lambda & & & \\ & \lambda & & \\ & & \ddots & \\ & & & \lambda \end{pmatrix}_{(K+1)\times(K+1)}$$

Where C represents customer arrival transitions.

$$A_0 = \begin{pmatrix} -(\lambda+\theta) & \theta & & & & \\ & -(\lambda+\theta) & \theta & & & \\ & & -(\lambda+\theta) & \theta & & \\ & & & \vdots & \vdots & \\ & & & & -(\lambda+\theta) & \theta \\ & & & & & -\lambda \end{pmatrix}_{(K+1)\times(K+1)}$$

The matrix B_1 represent service-related transitions, i.e., service

completion.

$$B_1=\begin{pmatrix} \eta+\alpha & & & & \\ & \eta+\alpha & & & \\ & & \eta+\alpha & & \\ & & & \ddots & \\ \mu & & & & \eta+\alpha \end{pmatrix}_{(K+1)\times(K+1)}$$

The matrix A_1 represents internal transitions when system has

customers (n≥1)

A_i=

$$\begin{pmatrix} -(\lambda+\eta+\theta+\alpha) & & & \\ & -(\lambda+\eta+\theta+\alpha) & & \\ & & \ddots & \\ & & & -(\lambda+\eta+\theta+\alpha) \end{pmatrix}_{(K+1)\times(K+1)} \quad \begin{matrix} \theta \\ \theta \\ \vdots \\ \theta \end{matrix}$$

$$B_i=\begin{pmatrix} \eta+\alpha & & & & \\ & \eta+\alpha & & & \\ & & \eta+\alpha & & \\ & & & \ddots & \\ & & & & \mu \end{pmatrix}_{(K+1)\times(K+1)}$$

Balance Equations

For each state

$(\lambda+\theta)P_{0,0} = (\eta+\alpha)P_{1,0} + \mu P_{1,k}$ (1)

State (0,0): No customers and server in firstvacation.

Leaving occurs because:

Arrival λ, vacation completion

Entering occurs from: θ

Service completion during vacation, normal service completion

$(\lambda+\theta+\eta+\alpha)P_{1,0} = \lambda P_{0,0} + (\eta+\alpha)P_{2,0}$ (2)

One customer during first vacation.

Possible events:

Arrival, service completion, reneging, vacation end

For $n \geq 2$,

$(\lambda+\theta+\eta+\alpha)P_{n,0} = \lambda P_{n-1,0} + (\eta+\alpha)P_{n+1,0}$, $\mathbf{n \geq 2,}$ **(3)**

Customers can arrive, leave (reneging)

For intermediate vacations (j=1,2,…K-1)

$(\lambda+\theta)P_{0,j} = (\eta+\alpha)P_{1,j} + \theta P_{0,j-1}$, 1≤j≤K-1, (4)

Move between vacations, customers arrive, service occurs slowly

Last Vacation to Normal Service:

$\lambda P_{0,K} = \theta P_{0,K-1}$, (5)

After the K^{th}vacation, server returns to normal state.

Normal Service State (j = K):

$(\lambda+\mu)P_{1,K} = \lambda P_{0,K} + \mu P_{2,K} + \theta \sum_{j=0}^{K-1} P_{1,j}$, (6)

Server working normally arrivals, service completion, vacation ending

In general,

$(\lambda+\mu)P_{n,K} = \lambda P_{n-1,K} + \mu P_{n+1,K} + \theta \sum_{j=0}^{K-1} P_{n,j}$, n≥2

Steady-State Analysis

The system is modelled as a continuous-time Markov chain (CTMC).

The steady-state probabilities are obtained using: $PQ = 0$ where,

P: steady-state probability vector

Q: generator matrix

Using the matrix geometric method, the solution is expressed as:

$P_n = P_0 R^n, n \geq 1$Type equation here.

where $R$is the rate matrix satisfying:

$R^2 B + RA + C = 0$Type equation here.

Stability Condition $\rho = \frac{\lambda}{\mu} < 1$

This ensures the existence of steady-state probabilities.

Performance Measures

The following performance measures are derived:

The performance of the proposed M/M/1 queue with variant working vacations and customer reneging is evaluated using steady-state probabilities obtained from the matrix geometric method. Let $P_{n,j}$denote the steady-state probability that there are $n$customers in the system and the server is in state j, where $j = 0,1,2,\ldots,K$.

Probability of Server in Working Vacation

The probability that the server is in a working vacation state is given by:

$$P_{WV} = \sum_{j=0}^{K-1} \sum_{n=0}^{\infty} P_{n,j}$$

This represents the fraction of time the server operates at a reduced service rate. A higher value indicates that the system frequently functions under lower efficiency.

Probability of Server in Busy State

The probability that the server is in the normal (regular) busy period is:

$$P_B = \sum_{n=1}^{\infty} P_{n,K}$$

This measures how often the server works at full capacity. Higher values of P_B indicate better system performance and faster service.

Expected Queue Length During Working Vacation

The expected number of customers in the system when the server is on working vacation is:

$$L_{WV} = \sum_{j=0}^{K-1} \sum_{n=0}^{\infty} n\, P_{n,j}$$

Since the service rate is reduced during vacation, this value is generally higher, reflecting increased congestion.

Expected Queue Length During Normal Service

The expected queue length during the regular busy period is:

$$L_B = \sum_{n=0}^{\infty} n\, P_{n,K}$$

This indicates the level of congestion when the server operates at full efficiency.

Probability of Empty System

The probability that there are no customers in the system is:

$$P_0 = \sum_{j=0}^{K} P_{0,j}$$

This measure reflects idle periods in the system. A higher value of P_0 indicates that the system is frequently empty, suggesting either low arrival rate or high service efficiency.

Customer Abandonment Rate (Reneging Rate)

Customers may leave the system due to impatience. The average abandonment rate is given by:

$$R_{reneging} = \alpha \sum_{n=1}^{\infty} n\, P_n \text{Type equation here.}$$

where:

$\alpha$is the reneging rate

$$P_n = \sum_{j=0}^{K} P_{n,j}$$

This measure quantifies the loss of customers due to waiting and is important in evaluating service quality.

Numerical Analysis

We consider the following values:

$\lambda = 1.67$(arrival rate)

$\mu = 2.5$(service rate)

$\eta = 0.59$(vacation service rate)

$\alpha = 0.62$(reneging rate)

θvaries

Traffic Intensity

$$\rho = \frac{\lambda}{\mu} = \frac{1.67}{2.5} = 0.668$$

Since $\rho < 1$, the system is stable.

Base Probability P_0

Using normalization:

$$\sum_{n=0}^{\infty} P_n = 1$$

Assuming geometric form:

$$P_n = P_0 \rho^n$$

Then:

$$P_0 = 1 - \rho = 1 - 0.668 = 0.332$$

Queue Length

$$L = \frac{\rho}{1-\rho} = \frac{0.668}{0.332} \approx 2.01$$

Effect of Vacation (Approximate Adjustment)

During vacation, service rate reduces to η:

$$\rho_{vac} = \frac{\lambda}{\eta} = \frac{1.67}{0.59} \approx 2.83$$

This shows heavy congestion during vacation; hence queue builds up.

Reneging Effect

Effective queue length reduces due to reneging:

$$L_{effective} = \frac{L}{1+\alpha}$$

$$L_{effective} = \frac{2.01}{1.62} \approx 1.24 \text{Type equation here.}$$

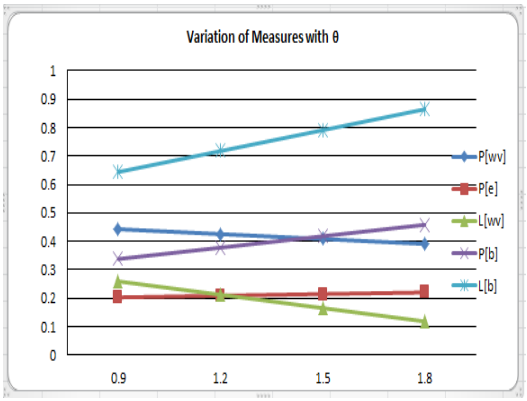
TABLE 1: Effect of Vacation Rate θ

θ	R_{WV}	P_B	L_{WV}	L_B	Total L
0.9	0.4428	0.3379	0.6440	0.2593	0.9033
1.2	0.4254	0.3780	0.7176	0.2124	0.9300
1.5	0.4081	0.4181	0.7911	0.1654	0.9565
1.8	0.3908	0.4583	0.8647	0.1185	0.9832

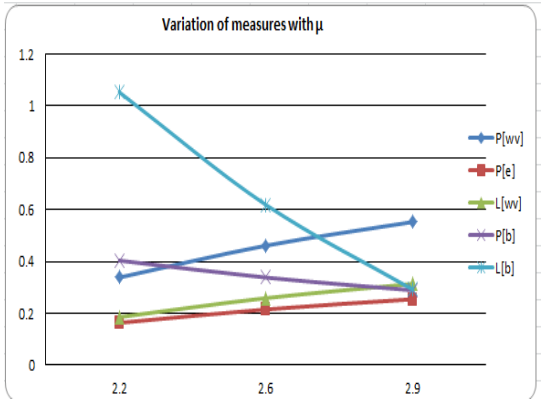
TABLE 2: Effect of Service Rate μ

μ	R_{WV}	P_B	L_{WV}	L_B	Total L
2.2	0.3380	0.4034	1.0539	0.1849	1.2388
2.6	0.4610	0.3379	0.6183	0.2584	0.8767
2.9	0.5524	0.2887	0.2916	0.3136	0.6052

Effect of θ on the measures:



Effect of μ on measures:



Conclusion

This study examined an M/M/1 queue with variant working vacations and customer reneging, providing a more realistic model of service systems where servers may operate at reduced capacity and customers may not wait indefinitely. Using a continuous-time Markov process and the matrix geometric method, steady-state probabilities and key performance measures were derived. The results show that system performance is strongly affected by service, vacation, and reneging rates, with higher service rates improving efficiency and reducing queue length. Overall, the model offers useful insights for designing and managing practical service systems.

REFERENCES

Laxmi, P. V., & Rajesh, P. (2020). Performance Characteristics of Discrete-Time Queue With Variant Working Vacations. *International Journal of Operations Research and Information Systems (IJORIS)*, 11(2), 1-21.

Laxmi, P. V., & Jyothsna, K. (2014). Performance analysis of variant working vacation queue with balking and reneging. *International Journal of Mathematics in Operational Research*, 6(4), 461-482.

Pikkala, V. L., &Edadasari, G. B. (2021). Variant working vacation Markovian queue with second optional service, unreliable server and retention of reneged customers. *International Journal of Mathematics in Operational Research*, 19(1), 45-64.

Vijaya Laxmi, P., Goswami, V., & Jyothsna, K. (2013). Analysis of finite buffer M/M/1 queue with balking, reneging and working vacations. *International Journal of Strategic Decision Sciences (IJSDS)*, 4(1), 1-24.

Vijaya Laxmi, P., & Jyothsna, K. (2016). Discrete-time impatient customer queue with Bernoulli-schedule vacation interruption. *Journal of Industrial Engineering International*, 12(2), 227-239.

Servi, L. D., & Finn, S. G. (2002). M/M/1 queues with working vacations (M/M/1/WV). *Performance Evaluation*,50(1),41-52.

Doshi, B. T. (1986). queuing systems with vacations—a survey. *queuing Systems*, 1(1), 29-66.

Kim, B., & Kim, J. (2014). A single server queue with variable service rates and vacations. *Operations Research Letters*, 42(3), 180-185.

Banik, A. D. (2009). The infinite buffer single server queue with a variant of multiple vacation policy and batch Markovian arrival process. *Applied Mathematical Modelling*, 33(7), 3025-3039.

Baba, Y. (2005). Analysis of GI/M/1 queue with multiple working vacations. *Operations Research Letters*, 33(2), 201-209.

Altman, E., &Yechiali, U. (2006). Analysis of customers' impatience in queues with server vacations. *queuing Systems*, 52(4), 261-279.

Yue, D., Zhang, Y., & Zhao, G. (2006). Analysis of an M/M/1 queue with balking, reneging and server vacations. *Proceedings of the 6th World Congress on Intelligent Control and Automation*, 1, 101-115.

Bouchentouf, A. A., &Medjahri, L. (2019). Performance and economic evaluation of differentiated multiple vacation queuing system with feedback and balked customers. *Applications and Applied Mathematics (AAM)*, 14(1), 46-62.

Selvaraju, N., & Goswami, C. (2013). Analysis of impatient customers in an M/M/1 queue with single and multiple working vacations. *Computers & Industrial Engineering*, 65(2), 207-215.

Haight, F. A. (1959). Queuing with reneging. *Metrika*, 2(1), 186-197.

Neuts, M. F. (1981).*Matrix-geometric solutions in stochastic models: An algorithmic approach*. Johns Hopkins University Press.

Zhang, Z. G., & Hou, Z. (2011). Analysis of a GI/M/1 queue with variant multiple working vacations. *Applied Mathematical Modelling*, 35(10), 4856-4866.

Do, T. V. (2010). M/M/1 retrial queue with working vacations. *Acta Informatica*, 47(1), 67-75.

Seenivasan, M., et al. (2022). Matrix-geometric method for queuing model with subject to breakdown and N-policy vacations. *Journal of Applied Mathematics and Computing*.

Manoharan, P., & Majid, S. (2025). Matrix-geometric analysis of heterogeneous server queuing systems with multiple working vacations. *DergiPark*.