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Research Article

DOMINATINGSETFORSOMEPARTICULAR GRAPHS

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Abstract

In graph theory, the concept of a dominating set plays a crucial role in understanding the structure and control of networks. A dominating set of a graph is a subset of vertices such that every vertex in the graph is either in this subset or adjacent to at least one vertex in it. This paper focuses on determining dominating sets for specific classes of graphs, analysing their properties, and identifying minimum dominating sets. Special attention is given to particular types of graphs such as paths, cycles, and complete graphs. The study explores methods to compute domination numbers and examines their applications in real-world problems like network design, resource allocation, and communication systems. The results contribute to a deeper understanding of optimization strategies in graph-based models.

INTRODUCTION

In Graph theory, study of domination is an important area that deals with selecting a subsetofverticesthatcaninfluenceorcontrolallotherverticesinagraph. The concept of dominating sets plays a significant role in understanding the structure and efficiency of networks.

A**graph**G=(V,E)consistsofasetofvertices VandasetofedgesEthatconnectpairsof vertices.In many real-life situations such as communication networks, transportation systems, and computer networks, it is necessary to determine a small group of important nodesthatcanmonitororreachallothernodesinthenetwork.Thisidealeadstotheconcept of dominating sets.

A **dominating set** of a graph G is a subset D of the vertex set V such that every vertex in thegraphiseitherinDoradjacenttoatleastonevertexinD.Inotherwords,eachvertexnot included in the dominating set must have at least one neighbor in the dominating set.This ensures that the vertices in D collectively “cover” or “control” the entire graph.

The smallest possible dominating set of a graph is of particular interest.The number of verticesinthissmallestdominatingsetiscalledthe**dominatingnumber** ofthegraphandis denoted by γ(G).Determining the domination number helps identifythe minimum number of vertices required to dominate the graph.

Dominating sets were first studied systematically in the 1950s and later became an important research topic in graph theory.Researchers have developed many results and theoremsrelatedtodominationindifferenttypesofgraphs.Theseresults helpunderstand how domination behaves in special graph structures.

Differenttypesof graphshavedifferentdominationnumbers.For example:

·InaCompleteGraph,asingle vertex candominatetheentiregraphbecauseevery vertex is connected to every other vertex

·InaCycleGraphandaPathGraph,verticesmustbechosencarefullysothatall vertices are covered efficiently.

·InaStarGraphandaWheelGraph,the centralvertexcandominate allother vertices.

Dominatingsetsandotherrelatedsubsetsandthecorrespondinggraphparametersfroman important research area of graph theory, which is rich is history, application, interesting results and unsolved research questions.This is the one of the fastest growing areas in graph areain graphtheoryand hasapplicationinsuchfieldsaslissarytocoveras linearalgebraand optimization design and analysis of communication networks, social science, computational complexity and algorithms design.

LetG =(V, X)beagraph.AsubsetSofViscalleddominatingsetifeveryvertex inV-S is adjacent to vertex in S.The minimum cardinality of a dominating set in G is called domination number of G and it is denoted by γ(G).The study of dominating sets in graph dates back to 1850’s with the study of the problem of determining the minimum number of queenswhicharenecessarytocoverann×n chessboard.Morethan50typesofdomination parameters have been studied by different authors.In 1850, chess freaks in Europe give through to issue for finding the board with a goal that each one of the blocks are either charged by a queen or in habited by a queen.The study of the dominating sets in graphs.

Alsogeneral asthe dominationofVerticesofagraph.

The study of domination in graphs was further developed in the late 1950's and 1960's, beginning with Claude Berge in 1958.Berge wrote a book on graph theory, in which he introducedthe"Coefficientofexternalstability",whichisnowasthedominationnumberof a graph.O systen are introduced the terms "dominatingset" and "domination number" in his book on graph theory which was published in 1962.The problems described above were studied in more detail around 1964 by brothers Yaglom and Yaglom.Their studies resulted instudiestosomeoftheseproblemfor books,kings,andbishops.Adecadelater,Cockayne and Hedetniemi published a survey paper, in which the notation (G) was first used for the domination numberof a graph G.Since this paper was published, domination in graphs has been studied extensively and several additional research papers have been published on this topic.

Graph theory is a wide area with more applications to real life.Domination in graphs is onethewell-studiedconceptsingraphtheory.Dominationsingraphhelpstheresearchesto get more ideas to manage the problem in the real-life.

Because of these theoretical importance and practical applications, domination has becomeanimportanttopicingraphtheory.Manyresearchescontinuetostudydominating sets in different types of graphs and explore new techniques to determine dominationnumbers.Inthisproject,dominatingsetsforsomeparticular graphssuchascompletegraphs, path graphs, cycle graphs, star graph, and wheel graphs are studied and their domination numbers are determined using known definitions and theorems.

Theorem: 3.1

LetSbeadominatingsetofagraphG.ThenSisminimaldominating setofGiffor every vetexu∈s one of the following conditions is satisfied

Uis a isolateofS ie,(N(u))⋂S=ϕ

Thereexistavertexv∈V-S(N(u))⋂S={u}).

Proof:

FirstwesupposethatSisminimaldominatingsetofG. Let u∈S.

ThenS-{u}isnotdominatingsetofG.Hencethere existv∈V-{S-{u}}=(V-U)∪{u} such that, (N(v))⋂{S-{u}}=ϕ.

Ifv=u,usatisfiescondition(i)Ifv≠uwethenhave(N(v))⋂{S{u}}=$\phi$and (N(v))⋂S≠

ϕ.

sinceS is a dominatingset.

Then(N(u))⋂S={u}andcondition(ii)satisfied.

Conversely,

LetussupposethatS isnot aminimaldominatingsetofG.

ie,thereexist S'⊆S suchthatS'is dominatingset ofG.Thenthereexist u∈S-S'.

ThenS'⊆S-{u}⊆S.

SinceS'isadominatingset,soisS-{u}.Thenasu∈S-{u},Itmustbeadjacenttoavertex in S-{u}.ie, u is not an isolate of S.

LetvØS,then(N(v))⋂{S-{u}=ϕie,(N(u))⋂S≠{u}.

Hence eithercondition(i)and (ii)issatisfied forthisparticularu∈S.

Theorem: 3.2

LetGbeagraphwithnoisolatedverticesandSbeaminimaldominatingsetofG.ThenV-S is also a dominating set of G.

Proof:

Let us consider that u∈S.The claim is N(u)¢S. LetussupposethatthecontrarywhereN(u)⊆S.

Thenumustbeadjacenttoavertex inSandanyvertex inV-Smust beadjacentonlyto vertex in S-{u}.

ThisimpliesS-{u}isadominatingset,acontradictionto minimalityofS. Then for any u∈S,N(u)¢S equivalently N(u)⋂V-S≠ ϕ.

⇒V-Sisdominatingset.

Theorem: 3.3

ForanygraphG,G'⊆CD(G)andG'=CD(G)iffeveryminimaldominatingsetofGis independent.

Proof:

Iff(uv)∈E(G'),thenextended{u,v}tomaximalindependentsetSofverticesin G. Since S is also a minimal dominating set of G, we obtain G'⊆CD(G).

Nowweprovesecondpart.

IfeveryminimaldominatingsetofGisindependent, thentwoverticesadjacentinG cannot be adjacent in CD(G).

ThusCD(G)⊆G'andtogether withG'⊆CD(G), weseethat CD(G)=G'.

Conversely,CD(G)⊆G'

⇒TwoverticesinsameminimaldominatingsetSarenotadjacentinG. That is , S is independent.

LetΔ(G)denotethemaximumdegreeofG.

Theorem: 3.4

Foranygraph withp vertices p≥2, CD(G)is connected iffΔ(G)<p-1

Proof:

LetΔ(G) <p-1 andu,vbeanytwoverticesof G.

If(u,v)ØE(G),thenbytheorem2.1,(u,v)∈CD(G).

If(u,v)∈E(G)andsomevertexwdistinctfrombothuandvisadjacent toneither,then again by theorem 3.3, in CD(G) the path uwv joins u to v.

Itonlyremainstoconsider(u,v)∈E(G)andeveryothervertexwisadjacenttoatleastone of u and v.

Then{u,v}isminimaldominatingsetofGandhence(u,v)∈CD(G).ThusCD(G)is connected.

Conversely,

SupposeCD(G)is connected.

Ifpossible,supposeΔ (G)=p-1anduisavertexofdegreep-1.Thenuisisolatedvertexin CD(G).

SinceGhas atleasttwo verticesCD(G)hasatleasttwo components,acontradiction.

ThusΔ(G)<p-1.

Note:

In a graph G,a vertex and an edge incident with it are said to cover each other.A set of verticeswhichcoversallthetheedgesisavertexcoverofG.Thevertexcoveringnumber

αo(G) in G is minimum number of vertices in a vertex cover.A set S of vertices in G is independentifnotwoverticesinSareadjacent.Theindependentnumber βo(G)ofGisthe maximum cardinality of an independent set ofvertices.

Theorem: 3.5

ForanygraphG,γCD(G)≤p-Γ(G)+1.

Proof:

LetS be aminimaldominatingsetwith|S|=Γ(G).

Thenγ(CD(G))≥ Γ(G).Thusγ(CD(G))≤p-Γ(G)+1followsfromthefactthat γ(G)≤p- Δ(G) and Δ(G)≤ ω(G)-1.

Foranygraph G, letαo(G) denotethevertexcoveringnumber of G.

Thatistheminimumnumberofverticesin avertexcoverandusethe fact that

αo(G)+βo(G)=p.

Theorem: 3.6

EverymaximalindependentsetofgraphGisminimaldominatingset.

Proof:

LetS be amaximaldominatingSet.ThenG must beadominatingset.

Ifnot,thereexistav∈V/SthatisnotdominatingbySandsoS∪{v}isan independentset of G, violating the maximality of S.

Further,Smust be aminimal dominatingsetofG.

Ifnot, there existavertexuofSsuchthatT=S/{u}is alsoadominatingsetofG.

ThismeansthatasunotbelongingtoT,uhas aneighborinTandhenceSisnot independent, a contradiction.

Hence everymaximalindependentsetofgraphGis minimaldominatingset.

Theorem: 3.7

Ina graphG,dominationnumberis lessthanorequalto(1,2)-dominationnumber.

Proof:

Let G be a graph and D beits dominatingset.Then everyvertex in V-D is adjacent to a vertex inD.Thatis,inD,foreveryvertex u,thereisavertex whichisatdistance1fromu.

Butitis notnecessarythat thereis asecondvertexatdistanceatmost 2from u.

Soifwefinda(1,2)-dominatingset,itwillcontainmoreverticesoratleastequalnumber of vertices than the dominating set.

Sothedominationnumberislessthanor equalto(1,2)-domination number.

Section3.8

Othervarietiesofdomination Paired Domination

PairedDominatingsetwhoseindirectsubgraphhasaperfectmatchingfromthedefnition it requires that there is no isolated vertices.EveryPaired Dominatingset is Dominatingset. Paired Domination number is minimum cardinality of Paired Dominating set and γpr.

K-Domination

AK-DominatingSetissetofverticesofDsuchthateachvertex inV(G)-Disdominated by atleast k vertices in D for fixed positive integer K.The minimum cardinality of k- dominating set is called k-Dominating number γk(G).

LocatingDomination

InwhichweinsistthateachvertexinV-DhasauniquesetofverticesD which dominate

it.

Distance Domination

InwhichweinsistthateachvertexinV-Dbewithindistancekofatleastonevertex inD, for a positive integer K.

Strong Domination

In a graph G=(V,E),a set D of vertices is said to be strong dominating set, for everyv in V-Dthereexistavertex uinDsuchthatuvϵE(G)anddegu¿=degv,thesubgraphofG.The cardinality of minimum strong dominating set is denoted by γs-set.

Distance2-Domination

AsetDisaDistance2-dominatingsetifforeveryvertex uϵV-D,d(u,D)<=2andis denoted by γ<=2(G).

TheDistance2-Dominationnumberγ<=2Gisequalstheminimumcardinalityofdistance 2-dominating set.

APPLICATIONOFDOMINATIONINGRAPH

APPLICATIONOFDOMINATIONINGRAPH

Domination in graphs has applications to several fields. Domination arises in facility location problems, where the number of facilities (e.g., hospitals, fire stations) is fixed and one attempts to minimize the distance that a person needs to travel to gettothe closest facility.Asimilarproblemoccurswhenthemaximumdistancetoafacilityisfixedandone attempts to minimize the number of facilities necessary so that everyone is serviced.

Concepts from domination also appear in problems involving finding sets of representatives, in monitoring communication or electrical networks, and in land surveying (e.g., minimizing thenumberofplacesasurveyormuststandinordertotakeheightmeasurementsforanentire region).

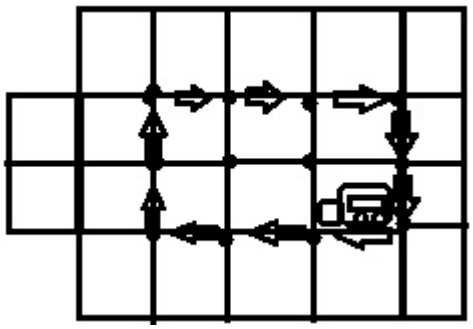
School bus routing

Mostschoolinthecountryprovideschoolbusesfortransportingchildrentoandfromschool Most also operate under certain rules, one of which usually states that no child shall have to walkfartherthan,sayonequarterkmtoabuspickuppoint.Thus,theymust constructaroute for each bus that gets within one quarter km of every child in its assigned area. No bus ride can take more than some specified number of minutes, and Limits on the number of children that a bus can carry at any one time. Let us say that the following figure represents a street map of part of a city, where each edge represents one pick up block. The school is located at the large vertex. Let us assume that the school has decided that no child shall have to walk morethan two blocks in order to bepicked up bya school bus. Construct a route for a school bus that leaves the school, gets within two blocks of every child and returns to the school

Locatingradarstationsproblem

TheproblemwasdiscussedbyBerge.Anumberofstrategiclocations aretobekept under surveillance. The goal is to locate a radar for the surveillance at as few of these locationsaspossible.Howasetoflocationsinwhichtheradarstationsaretobeplacedcan be determined

Nuclearpowerplants problem



z similar known problem is a nuclear power plants problem. There are various locations and an arc can be drawn from location x to location y if it is possible for a watchman stationed at x to observe a warning light located at y. How many guards are needed toobserveallofthewarninglights,andwhereshouldtheybelocated? Atpresent,dominationis considered to be one of the fundamental concepts in graph theoryand its various applications to ad hoc networks, biological networks, distributed computing, social networks and web graphs [1, 25, 27, 47] partly explain the increased interest. Such applications usually aim to select a subset of nodes that will provide some definite service such that every node in the network is close to some node in the subset. The following examples show when the concept of domination can be applied in modeling real-life problems.

Modelingbiologicalnetworks

Usinggraphtheoryasamodelingtoolinbiologicalnetworksallowstheutilizationofthe most graphical invariants in such a way that it is possible to identify secondary RN(Ribonucleic acid) motifs numerically.Those graphical invariants are variations of the domination number of a graph.

Theresultsoftheresearchcarriedoutinshowthatthevariationsofthe domination number can be used for correctly distinguishing among the trees that represent native structures and those that are not likely candidates to represent RNA.

Modelingsocialnetworks

Dominating sets can be used in modeling social networks and studying the dynamics of relations among numerous individuals in different domains. A social network is a social structuremadeofindividuals(orgroupsofindividuals),whichareconnectedbyoneormore specific types of interdependency. The choice of initial sets of target individuals is an important problem in the theory of social networks. In the work of Kelleher and Cozzens, social networks are modeled in terms of graph theory and it was shown that some of these sets can be found by using the properties of dominating sets in graphs.

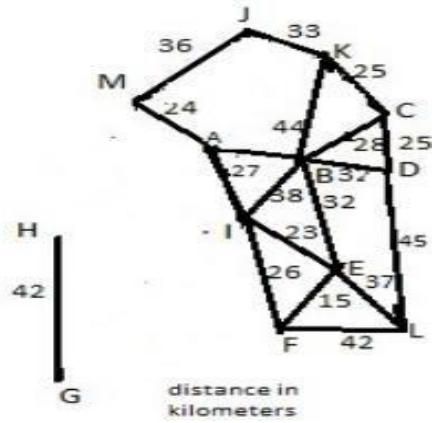
Facilitylocationproblem

The dominating sets in graphs are natural models for facility location problems in operational research. Facility location problems are concerned with the location of one or morefacilitiesinawaythatoptimizesacertainobjectivesuchasminimizingtransportation cost, providing equitable service to customers and capturing the largest market share.

Computercommunicationnetwork

ConsideracomputernetworkmodeledbyagraphG=(V,E),forwhichverticesrepresents computers and edges represent direct links between pairs of computers. Let the vertices in following figure

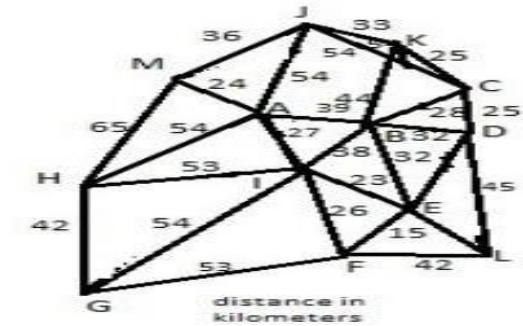
represent an array, or network, of 16 computers, or processors. Each processor to which it is directly connected. Assume that from time to time we need to collect information from all processors. We do this byhavingeach processor route its information to one of a small set of collecting processors (a dominating set). Since this



must be done relatively fast, we cannot route this information over too long a path. Thus we identify asmall set ofprocessors which are close to all other processors. Let us saythat wewill tolerate at most a two unit delay between the time a processor sends its information and the time it arrives at a nearby collector. In this case we seek a distance-2 dominating set among the setof all processors. The two shaded vertices form a distance-2 dominating set in the hypercube network in following figure

Radio stations

Suppose that we have a collection of small villages in a remote part of the world. We wouldliketolocateradiostationsinsomeofthesevillagessothatmessage scanbebroadcast to all of the villages in the region. Since each radio station has a limited broadcasting range, wemust use several stations to reach all villages. But since radio stations are costly, wewant tolocateasfewaspossiblewhichcanreachallothervillages.Leteachvilla geberepresented by a vertex. An edge between two villages is labeled with the distance, say in kilometers, between the two villages

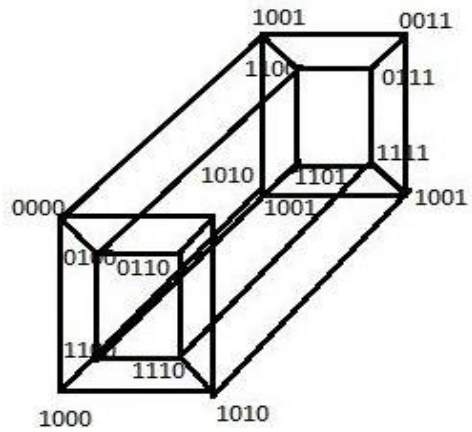


Let us assume that a radio stations has a broadcast range of fifty kilometes.What is the leastnumberofstationsinasetwhichdominateswithindistance50alloth er inthisgraph? A set B,F,H,J of cardinality four is indicatedinthefigure2

Here we have assumed that a radio station hasa broadcast range of only fifty kilometres,we can essentially remove all edges in the graph, which represent a distance of morethanfiftykilometers.Weneedonlytofind adominatingsetinthisgraph.Noticethatifwecouldaffordradiostations whichhaveabroadcastrangeofseventykilometers,three radio stations would sufficient.

Coding theory

Theconceptofdominationisalsoappliedincodingtheoryasdiscusse d byKalbfleisch, Stanton and Horton and Cockayne and Hedetniemi.If one defines a graph, the vertices of which are the n-dimensional vectors with coordinates chosen from 1,..., p, p >1, and two vertices are adjacent if they differ in one coordinate, then



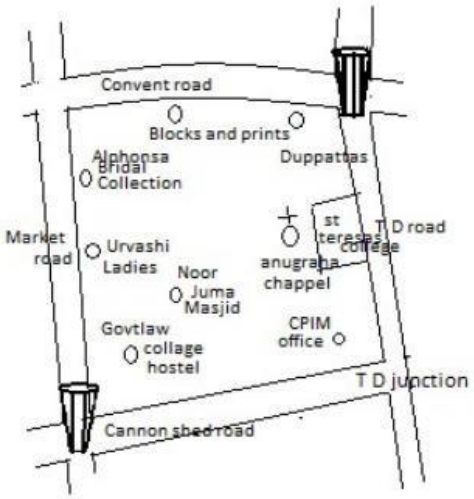
the sets of vectors which are (n,p)coveringsets,singleerrorcorrectingcodes,orperfectcoveringsetsa realldominating sets of the graph with determined additional properties.

Multipledomination problem

An important role is played by multiple domination. Multiple domination can be used to construct hierarchical overlay networks in peer-to-peer applications for more efficient index searching.Thehierarchicaloverlaynetworks usuallyserveasdistributeddatabasesforindex searching, e.g. in modern file sharing and instant messaging computer network applications. Dominatingsetsofseveralkindsareusedforbalancingefficiencyandfau lttoleranceaswell as in the distributed construction of minimum spanning trees. Another good example of direct, important and

quickly developing applications of multiple domination in modern computer networks is a wireless sensor network.

?Howmanywastebinscanbeplacedtoreducepollutioningivenmap(convent junction)?



CONCLUSION:

The main aim of this project is to present the importance of graph theoretical ideas in various areas of a science and engineering for domination in graphs theoretical concepts for the research.An overview is presented especially to project the idea of graph theory.so, the graph theory section of each paper is given importance than to other sections.Researches may get some information related to graph theory and its application some ideas related to their research field. In graph theory, there are many stability parameters such as vertex domination number, independence number etc.The domination number of a graph is a new vulnerability measure that considers the neighbourhood of vertices.From the definition of domination, ever vertex of a graph must be protected byits neigh bourhood.In this search, the main ideas is each u,v∈V must be protected and are capable of dominating of both u and v. Alsowediscussedabouttotaldomination,independentdomination,(1,2)dominationindetail and also discussed varieties of domination and application of domination in different fields.

The concept of domination is an important topic in graph theory that provides valuable insight into the control and coverage of networks.he study of dominating sets in different typesofgraphshelpsdevelopadeeperunderstandingofgraphstructuresa ndtheirproperties. As research in graph theory continues to grow, dominating sets will remain a key area of study with both theoretical significance and practical revelance in many scientific and technological fields.

REFERENCE

GierAgnarSon,RaymondGreenLaw,Graphtheorymodelling,applicati onsand algorithms.

T.TamizhChelvamandBJayaprasad,Onindependentdomination number.

RBalakrishnan,R.J.Wilson, GSethuraman-Graphtheoryandits application.

GSureshsingh,Graphtheory.

Ping-ZhuDu,Ping-JunWan,Connecteddominatingset;Theoryand applications.

VSwaminathanandS.V.Padmavathi,structuralstrongdominationof graphs.

TerasaWHaynes,StephenTHedetniemiandPeterJSlater,Fundamenta lsofdomination in graphs.

MeeraPaulsonandLillyTI,Dominationingraphtheory.

DavidAmos,Universityof HoustonDowntown, On totaldomination in graphs.

NadiaNosratiKenareh,Dominationin graphs.

VRKulliandBJanakiram,thecommonminimaldominating graph.

K.AmeenalBibi,ALakshmiandRJothilakshmi,Applicationsofdistance2 -dominating set of graph network.

A.Sasireka,A.HNandhuKishore,Applicationsofdominatingsetofgrap hinComputer Networks.

Irvingh,R.W.(1991),"Onapproximatingtheminimumindependentdo minatingset".

Goddard,Waryne;Henning,MichaelA.(2013),Independentdomianat ioningraphs.

JenniferMTarr,Dominationingraphs,(2010).

PreetiGupta,Dominationingraphwithapplication,IndianJ.Res2(2013),no.3,115- continues to grow, dominating sets will remain a key area of study with both theoretical significance and practical revelance in many scientific and technological fields.