

LouisSaveinDupuisJournalofMultidisciplinaryResearch2024,  
3:313-320 Doi.: 10.21839/lsdjmr.2024.v3.198

Research Article

## Prime Labeling Of Goldner Harary Graph

K. Bharatha Devi,

Research Scholar Department of Mathematics,

Arignar Anna Government Arts College, Villupuram.

Dr.S. Lakshmi Narayanan,

Associate. Professor & Head of department,

Arignar Anna Government Arts College, Villupuram.

### Abstract

A graph  $G = (V(G), E(G))$  is observed to admit prime labeling, if a graph that receives prime labeling is called prime graph. In this research article we investigate that the Goldner Harary graph admits prime labeling. We establish prime labeling using some graph operations such as duplication. Switching and fusion with few ideas.

Key words: *Goldner Harary graph, switching, duplication, fusion.*

### Introduction

In this we define a connected and undirected graph name **Goldner Harary** graph and we denote the vertex set by  $V(G)$  and edge set by  $E(G)$  of graph  $G$  and their corresponding cardinality by  $|V(G)|$  and  $|E(G)|$ . Here we establish that **Goldner Harary** graph admits prime labeling.

### Definition

In the mathematical field of graph theory, the goldnerharary graph is a simple undirected graph with 11 vertices and 27 edges. It is named after a goldner and frank harary, who proved in 1975 that it was the smallest non Hamiltonian maximal planer graph.

### Theorem: 1

The goldner harary graph  $G$  is a prime graph

### Proof:

Given  $G$  be a goldner harary graph

Let  $v_1, v_2, \dots, v_{11}$  vertices of  $G$

$V(G) = \{v_1, v_2, \dots, v_{11}\}$ ,  $E(G) = \{e_1, e_2, \dots, e_{27}\}$

$|V(G)| = 11$   $|E(G)| = 27$

Now we define the function  $f: V(G) \rightarrow \{1, 2, \dots, 11\}$

such that  $f(v_i) = i$ ,  $i = \{1, 2, 3, 5, 7, 9, 11\}$

$f(v_i) = 3i + 1$ ,  $i = \{4, 6, 8, 10\}$  by  $f(v_1) = 1$ ,  $f(v_2) = 2$ ,  
 $f(v_3) = 3$ ,  $f(v_4) = 13$ ,  $f(v_5) = 5$ ,  $f(v_6) = 19$ ,  $f(v_7) = 7$ ,  
 $f(v_8) = 25$ ,  $f(v_9) = 9$ ,

$f(v_{10}) = 31$ ,  $f(v_{11}) = 11$

The relative prime of adjacent vertices have to be verify

We look at the following types of edges:

$\text{Gcd}\{f(v_i), f(v_{i+1})\} = 1$  for  $i = 1, 2, 3, 4, 5, 6, 7$ ,

$\text{Gcd}\{f(v_8), f(v_2)\} = 1$

$\text{Gcd}\{f(v_1), f(v_4)\} = 1$

$\text{Gcd}\{f(v_2), f(v_9)\} = 1$

$\text{Gcd}\{f(v_9), f(v_4)\} = 1$

$\text{Gcd}\{f(v_4), f(v_{10})\} = 1$

$\text{Gcd}\{f(v_{10}), f(v_6)\} = 1$

$\text{Gcd}\{f(v_1), f(v_{10})\} = 1$

$\text{Gcd}\{f(v_1), f(v_{11})\} = 1$

$\text{Gcd}\{f(v_1), f(v_8)\} = 1$

$\text{Gcd}\{f(v_7), f(v_3)\} = 1$

$\text{Gcd}\{f(v_4), f(v_6)\} = 1$

$\text{Gcd}\{f(v_2), f(v_4)\} = 1$

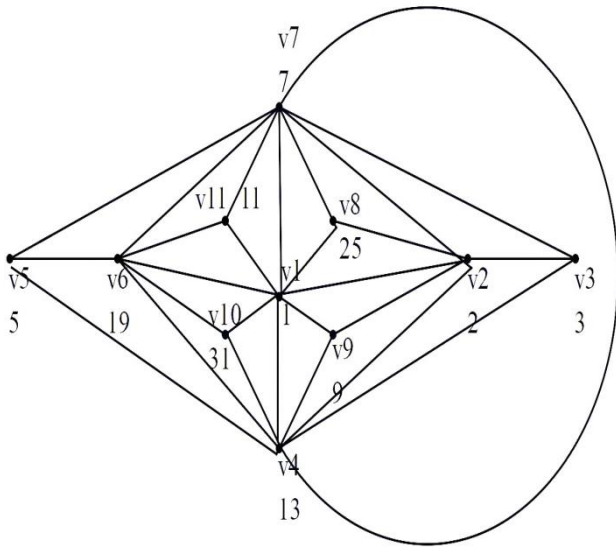
$\text{Gcd}\{f(v_{11}), f(v_6)\} = 1$

$\text{Gcd}\{f(v_7), f(v_1)\} = 1$

$\text{Gcd}\{f(v_1), f(v_9)\} = 1$

$\text{Gcd}\{f(v_7), f(v_4)\} = 1$

$$\begin{aligned} \text{Gcd}\{f(v_7),f(v_5)\}&=1 \\ \text{Gcd}\{f(v_7),f(v_6)\}&=1 \\ \text{Gcd}\{f(v_7),f(v_{11})\}&=1 \end{aligned}$$



As a result,  $f$  meets the prime labeling condition. Hence Goldner Harary accept prime labeling.

**Theorem: 2**

In Goldner harary graph the duplication of any vertex allows prime labelling

**Proof:**

Consider the Goldnerharary graph which has 11 vertices 27 edges.

**Case: 1**

Let  $G$  be the goldnerharary graph generated by duplicating any vertex

We can consider  $v_1$  to be the duplicating vertex, and let  $v_{1'}$  be the duplication vertex of  $v_1$

Given  $G$  be a goldner harary graph

let  $v_1', v_1, v_2, \dots, v_{11}$  vertices of  $G$

$$V(G) = \{ v_1', v_1, v_2, \dots, v_{11} \}, E(G) = \{ e_1, e_2, \dots, e_{35} \}$$

$$|V(G)| = 12 \quad |E(G)| = 35$$

Now we define the function  $f: V(G) \rightarrow \{1, 2, \dots, 11, 12\}$

$$\text{such that } f(v_i) = i, \quad i = \{1, 2, 3, 5, 7, 9, 11\}$$

$$f(v_i) = 3i + 1, \quad i = \{4, 6, 8, 10\} \text{ by}$$

$$f(v_{1'}) = 3i + 1, \quad i = 12$$

Which implies  $f(v_{1'}) = 37$

$$f(v_1) = 1, f(v_2) = 2, f(v_3) = 3, f(v_4) = 13, f(v_5) = 5, f(v_6) = 19, f(v_7) = 7, f(v_8) = 25, f(v_9) = 9,$$

$$f(v_{10}) = 31, f(v_{11}) = 11$$

The relative prime of adjacent vertices have to be verify

We look at the following types of edges:

$$\text{Gcd}\{f(v_i), f(v_{i+1})\} = 1 \quad \text{for } i = 1, 2, 3, 4, 5, 6, 7,$$

$$\text{Gcd}\{f(v_8), f(v_2)\} = 1$$

$$\text{Gcd}\{f(v_1), f(v_4)\} = 1$$

$$\text{Gcd}\{f(v_2), f(v_9)\} = 1$$

$$\text{Gcd}\{f(v_9), f(v_4)\} = 1$$

$$\text{Gcd}\{f(v_4), f(v_{10})\} = 1$$

$$\text{Gcd}\{f(v_{10}), f(v_6)\} = 1$$

$$\text{Gcd}\{f(v_1), f(v_{10})\} = 1$$

$$\text{Gcd}\{f(v_1), f(v_{11})\} = 1$$

$$\text{Gcd}\{f(v_1), f(v_8)\} = 1$$

$$\text{Gcd}\{f(v_7), f(v_3)\} = 1$$

$$\text{Gcd}\{f(v_4), f(v_6)\} = 1$$

$$\text{Gcd}\{f(v_{11}), f(v_6)\} = 1$$

$$\text{Gcd}\{f(v_7), f(v_1)\} = 1$$

$$\text{Gcd}\{f(v_1), f(v_9)\} = 1$$

$$\text{Gcd}\{f(v_7), f(v_4)\} = 1$$

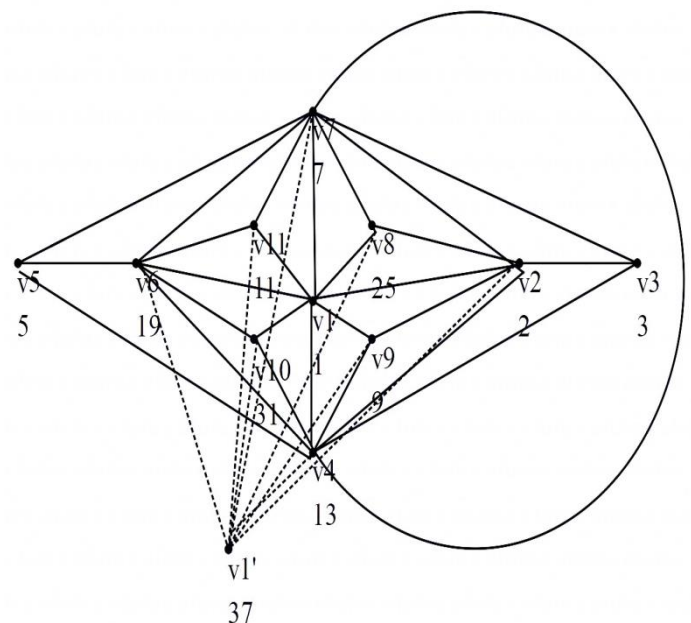
$$\text{Gcd}\{f(v_7), f(v_5)\} = 1$$

$$\text{Gcd}\{f(v_7), f(v_6)\} = 1$$

$$\text{Gcd}\{f(v_7), f(v_{11})\} = 1$$

$$\text{Gcd}\{f(v_2), f(v_4)\} = 1$$

$$\text{Gcd}\{f(v_i), f(v_{i'})\} = 1, \quad i = 2, 4, 6, 7, 8, 9, 10, 11$$



As a result  $G$  admits prime labelling.

Hence  $G$  is a prime graph

**Case: 2**

Let  $G$  be the goldnerharary graph generated by duplicating any vertex

We can consider  $v_7$  to be the duplicating vertex, and let  $v_7'$  be the duplication vertex of  $v_7$

Given  $G$  be a goldner harary graph

let  $v_7', v_1, v_2, \dots, v_{11}$  vertices of  $G$

$V(G) = \{ v_7', v_1, v_2, \dots, v_{11} \}$ ,  $E(G) = \{ e_1, e_2, \dots, e_{35} \}$

$|V(G)| = 12$   $|E(G)| = 35$

Now we define the function  $f: V(G) \rightarrow \{1, 2, \dots, 11, 12\}$

such that  $f(v_i) = i$ ,  $i = \{1, 2, 3, 5, 7, 9, 11\}$   $f(v_i) = 3i + 1$ ,

$i = \{4, 6, 8, 10\}$  by

$f(v_{7'}) = 3i + 1, i = 12$

Which implies  $f(v_{7'}) = 37$

$f(v_1) = 1, f(v_2) = 2, f(v_3) = 3, f(v_4) = 13, f(v_5) = 5,$   
 $f(v_6) = 19, f(v_7) = 7, f(v_8) = 25, f(v_9) = 9,$

$f(v_{10}) = 31, f(v_{11}) = 11$

The relative prime of adjacent vertices have to be verify

We look at the following types of edges:

$\text{Gcd}\{f(v_i), f(v_{i+1})\} = 1$  for  $i = 1, 2, 3, 4, 5, 6, 7,$

$\text{Gcd}\{f(v_8), f(v_2)\} = 1$

$\text{Gcd}\{f(v_1), f(v_4)\} = 1$

$\text{Gcd}\{f(v_2), f(v_9)\} = 1$

$\text{Gcd}\{f(v_9), f(v_4)\} = 1$

$\text{Gcd}\{f(v_4), f(v_{10})\} = 1$

$\text{Gcd}\{f(v_{10}), f(v_6)\} = 1$

$\text{Gcd}\{f(v_1), f(v_{10})\} = 1$

$\text{Gcd}\{f(v_1), f(v_{11})\} = 1$

$\text{Gcd}\{f(v_1), f(v_8)\} = 1$

$\text{Gcd}\{f(v_7), f(v_3)\} = 1$

$\text{Gcd}\{f(v_4), f(v_6)\} = 1$

$\text{Gcd}\{f(v_{11}), f(v_6)\} = 1$

$\text{Gcd}\{f(v_7), f(v_1)\} = 1$

$\text{Gcd}\{f(v_1), f(v_9)\} = 1$

$\text{Gcd}\{f(v_7), f(v_4)\} = 1$

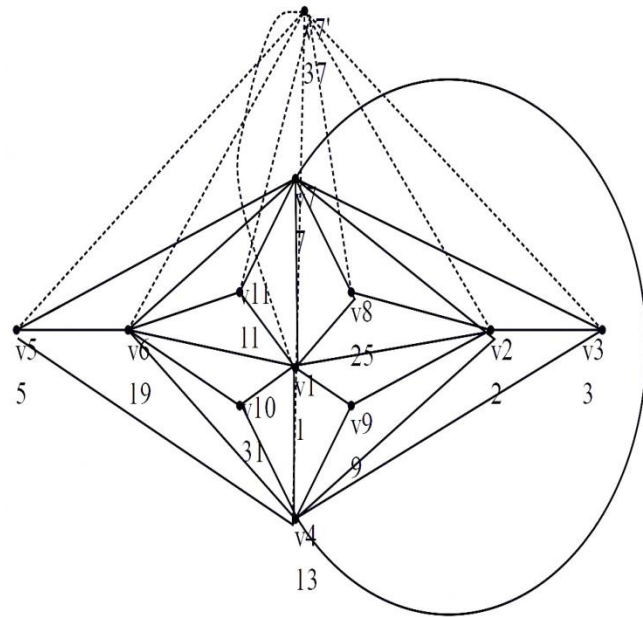
$\text{Gcd}\{f(v_7), f(v_5)\} = 1$

$\text{Gcd}\{f(v_7), f(v_6)\} = 1$

$\text{Gcd}\{f(v_7), f(v_{11})\} = 1$

$\text{Gcd}\{f(v_2), f(v_4)\} = 1$

$\text{Gcd}\{f(v_i), f(v_{7'})\} = 1, i = 1, 2, 3, 4, 5, 6, 8, 11$



As a result  $G$  admits prime labelling.

Hence  $G$  is a prime graph.

**Case: 3**

Let  $G$  be the goldnerharary graph generated by duplicating any vertex

We can consider  $v_{11}$  to be the duplicating vertex, and let  $v_{11}'$  be the duplication vertex of  $v_{11}$

given  $G$  be a goldnerharary graph

let  $v_{11}', v_1, v_2, \dots, v_{11}$  vertices of  $G$

$V(G) = \{ v_{11}', v_1, v_2, \dots, v_{11} \}$ ,  $E(G) = \{ e_1, e_2, \dots, e_{30} \}$

$|V(G)| = 12$   $|E(G)| = 30$

Now we define the function  $f: V(G) \rightarrow \{1, 2, \dots, 11, 12\}$

such that  $f(v_i) = i, i = \{1, 2, 3, 5, 7, 9, 11\}$

$f(v_i) = 3i + 1, i = \{4, 6, 8, 10\}$  by

$f(v_{11}') = 3i + 1, i = 12$

which implies  $f(v_{11}') = 37$

$f(v_1) = 1, f(v_2) = 2, f(v_3) = 3, f(v_4) = 13, f(v_5) = 5,$   
 $f(v_6) = 19, f(v_7) = 7, f(v_8) = 25, f(v_9) = 9,$

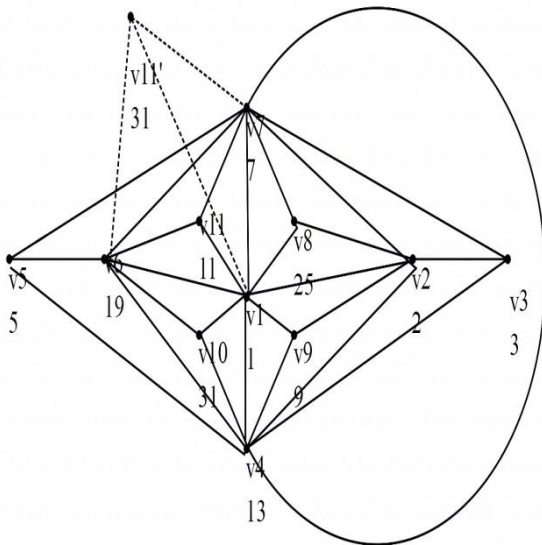
$f(v_{10}) = 31, f(v_{11}) = 11$

The relative prime of adjacent vertices have to be verify

We look at the following types of edges:

$\text{Gcd}\{f(v_i), f(v_{i+1})\} = 1$  for  $i = 1, 2, 3, 4, 5, 6, 7,$

- Gcd{ f(v<sub>8</sub>),f(v<sub>2</sub>)}=1
- Gcd{f(v<sub>1</sub>),f(v<sub>4</sub>)}=1
- Gcd{f(v<sub>2</sub>),f(v<sub>9</sub>)}=1
- Gcd{f(v<sub>9</sub>),f(v<sub>4</sub>)}=1
- Gcd{f(v<sub>4</sub>),f(v<sub>10</sub>)}=1
- Gcd{f(v<sub>10</sub>),f(v<sub>6</sub>)}=1
- Gcd{f(v<sub>1</sub>),f(v<sub>10</sub>)}=1
- Gcd{f(v<sub>1</sub>),f(v<sub>11</sub>)}=1
- Gcd{f(v<sub>1</sub>),f(v<sub>8</sub>)}=1
- Gcd{f(v<sub>7</sub>),f(v<sub>3</sub>)}=1
- Gcd{f(v<sub>4</sub>),f(v<sub>6</sub>)}=1
- Gcd{f(v<sub>11</sub>),f(v<sub>6</sub>)}=1
- Gcd{f(v<sub>7</sub>),f(v<sub>1</sub>)}=1
- Gcd{f(v<sub>1</sub>),f(v<sub>9</sub>)}=1
- Gcd{f(v<sub>7</sub>),f(v<sub>4</sub>)}=1
- Gcd{f(v<sub>i</sub>), (v<sub>11'</sub>)}=1 , i=1,6,7



As a result G admits prime labelling.

Hence G is a prime graph

**Theorem: 3**

The graph obtained by switching of a vertex v<sub>1</sub> in a Goldner harary graph admits prime labeling.

**Proof:**

Consider the prism graph which has 11 vertices and 27 edges

**Case: 1**

Let G<sub>u</sub> denotes the graph obtained by vertex switching of Goldner harary graph with respect to the vertex V<sub>1</sub>

|V(G<sub>u</sub>)|=11 |E(G<sub>u</sub>)|=21

Now we define the function f:V(G) → {1,2,...,11}

such that f(v<sub>i</sub>)=i , i={1,2,3,5,7,9,11}

f(v<sub>i</sub>)=3i+1 , i={4,6,8,10} by

f(v<sub>1</sub>)=1 , f(v<sub>2</sub>)=2 ,f(v<sub>3</sub>)=3 , f(v<sub>4</sub>)=13 , f(v<sub>5</sub>)=5 , f(v<sub>6</sub>)=19 , f(v<sub>7</sub>)=7 , f(v<sub>8</sub>)=25 , f(v<sub>9</sub>)=9,

f(v<sub>10</sub>)=31, f(v<sub>11</sub>)=11

The relative prime of adjacent vertices have to be verify.

We look at the following types of edges:

Gcd{f(v<sub>i</sub>),f(v<sub>i+1</sub>)}=1 for i=2,3,4,5,6,7,

Gcd{ f(v<sub>8</sub>),f(v<sub>2</sub>)}=1

Gcd{f(v<sub>2</sub>),f(v<sub>9</sub>)}=1

Gcd{f(v<sub>9</sub>),f(v<sub>4</sub>)}=1

Gcd{f(v<sub>4</sub>),f(v<sub>10</sub>)}=1

Gcd{f(v<sub>10</sub>),f(v<sub>6</sub>)}=1

Gcd{f(v<sub>7</sub>),f(v<sub>3</sub>)}=1

Gcd{f(v<sub>4</sub>),f(v<sub>6</sub>)}=1

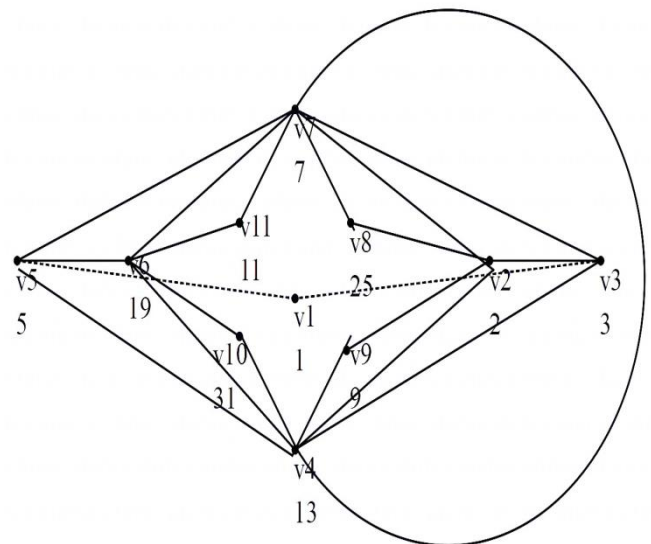
Gcd{f(v<sub>11</sub>),f(v<sub>6</sub>)}=1

Gcd{f(v<sub>3</sub>),f(v<sub>1</sub>)}=1

Gcd{f(v<sub>5</sub>),f(v<sub>1</sub>)}=1

Gcd{f(v<sub>7</sub>),f(v<sub>4</sub>)}=1

Gcd{f(v<sub>2</sub>),f(v<sub>4</sub>)}=1



Thus f is a prime labeling and consequently G<sub>u</sub> is prime graph.

Therefore the switching of a vertex v<sub>1</sub> in a Goldner Harary graph admits prime labeling.

**Case: 2**

Let  $G_u$  denotes the graph obtained by vertex switching of Goldner Harary graph with respect to the vertex  $V_7$

$$|V(G_u)|=11 \quad |E(G_u)|=21$$

Now we define the function  $f:V(G_u) \rightarrow \{1,2,\dots,11\}$

$$\text{such that } f(v_i)=i \quad , \quad i=\{1,2,3,5,7,9,11\}$$

$$f(v_i)=3i+1 \quad , \quad i=\{4,6,8,10\} \text{ by}$$

$$f(v_1)=1 \quad , \quad f(v_2)=2 \quad , \quad f(v_3)=3 \quad , \quad f(v_4)=13 \quad , \quad f(v_5)=5 \quad ,$$

$$f(v_6)=19 \quad , \quad f(v_7)=7 \quad , \quad f(v_8)=25 \quad , \quad f(v_9)=9 \quad ,$$

$$f(v_{10})=31 \quad , \quad f(v_{11})=11$$

The relative prime of adjacent vertices have to be verify

We look at the following types of edges:

$$\text{Gcd}\{f(v_i),f(v_{i+1})\}=1 \quad \text{for } i=1,2,3,4,5,6,$$

$$\text{Gcd}\{f(v_8),f(v_2)\}=1$$

$$\text{Gcd}\{f(v_1),f(v_4)\}=1$$

$$\text{Gcd}\{f(v_2),f(v_9)\}=1$$

$$\text{Gcd}\{f(v_9),f(v_4)\}=1$$

$$\text{Gcd}\{f(v_4),f(v_{10})\}=1$$

$$\text{Gcd}\{f(v_{10}),f(v_6)\}=1$$

$$\text{Gcd}\{f(v_1),f(v_{10})\}=1$$

$$\text{Gcd}\{f(v_1),f(v_{11})\}=1$$

$$\text{Gcd}\{f(v_1),f(v_8)\}=1$$

$$\text{Gcd}\{f(v_4),f(v_6)\}=1$$

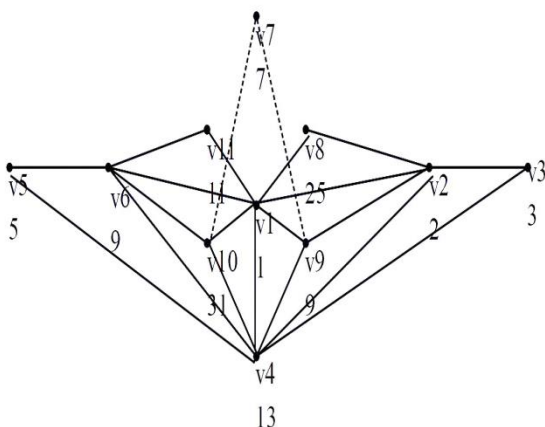
$$\text{Gcd}\{f(v_{11}),f(v_6)\}=1$$

$$\text{Gcd}\{f(v_7),f(v_{10})\}=1$$

$$\text{Gcd}\{f(v_1),f(v_9)\}=1$$

$$\text{Gcd}\{f(v_7),f(v_9)\}=1$$

$$\text{Gcd}\{f(v_2),f(v_4)\}=1$$



Thus  $f$  is a prime labeling and consequently  $G_u$  is prime graph.

Therefore the switching of a vertex  $v_7$  in a Goldner Harary graph admits prime labeling.

**Case: 3**

Let  $G_u$  denotes the graph obtained by vertex switching of Goldner Harary graph with respect to the vertex  $V_{11}$

$$|V(G_u)|=11 \quad |E(G_u)|=32$$

Now we define the function  $f:V(G) \rightarrow \{1,2,\dots,11\}$

$$\text{such that } f(v_i)=i \quad , \quad i=\{1,2,3,5,7,9,11\}$$

$$f(v_i)=3i+1 \quad , \quad i=\{4,6,8,10\} \text{ by}$$

$$f(v_1)=1 \quad , \quad f(v_2)=2 \quad , \quad f(v_3)=3 \quad , \quad f(v_4)=13 \quad , \quad f(v_5)=5 \quad ,$$

$$f(v_6)=19 \quad , \quad f(v_7)=7 \quad , \quad f(v_8)=25 \quad , \quad f(v_9)=9 \quad ,$$

$$f(v_{10})=31 \quad , \quad f(v_{11})=11$$

The relative prime of adjacent vertices have to be verify

We look at the following types of edges:

$$\text{Gcd}\{f(v_i),f(v_{i+1})\}=1 \quad \text{for } i=1,2,3,4,5,6,7,$$

$$\text{Gcd}\{f(v_8),f(v_2)\}=1$$

$$\text{Gcd}\{f(v_1),f(v_4)\}=1$$

$$\text{Gcd}\{f(v_2),f(v_9)\}=1$$

$$\text{Gcd}\{f(v_9),f(v_4)\}=1$$

$$\text{Gcd}\{f(v_4),f(v_{10})\}=1$$

$$\text{Gcd}\{f(v_{10}),f(v_6)\}=1$$

$$\text{Gcd}\{f(v_1),f(v_{10})\}=1$$

$$\text{Gcd}\{f(v_8),f(v_{11})\}=1$$

$$\text{Gcd}\{f(v_1),f(v_8)\}=1$$

$$\text{Gcd}\{f(v_7),f(v_3)\}=1$$

$$\text{Gcd}\{f(v_4),f(v_6)\}=1$$

$$\text{Gcd}\{f(v_2),f(v_4)\}=1$$

$$\text{Gcd}\{f(v_{11}),f(v_2)\}=1$$

$$\text{Gcd}\{f(v_7),f(v_1)\}=1$$

$$\text{Gcd}\{f(v_1),f(v_9)\}=1$$

$$\text{Gcd}\{f(v_7),f(v_4)\}=1$$

$$\text{Gcd}\{f(v_7),f(v_5)\}=1$$

$$\text{Gcd}\{f(v_7),f(v_6)\}=1$$

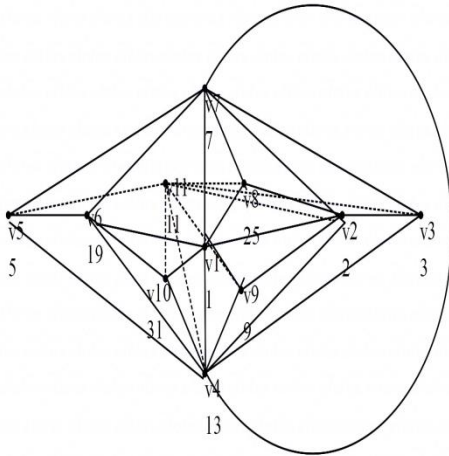
$$\text{Gcd}\{f(v_9),f(v_{11})\}=1$$

$$\text{Gcd}\{f(v_{11}),f(v_4)\}=1$$

$$\text{Gcd}\{f(v_{11}),f(v_{10})\}=1$$

$$\text{Gcd}\{f(v_5),f(v_{11})\}=1$$

$$\text{Gcd}\{f(v_3),f(v_{11})\}=1$$



Thus  $f$  is a prime labeling and consequently  $G_u$  is prime graph.

Therefore the switching of a vertex  $v_{11}$  in a Goldner Harary graph admits prime labeling.

**Theorem: 4**

The graph obtained by fusing of vertices  $v_7$  and  $v_3$  in a Goldner Harary admits prime labeling.

**Proof:**

Consider Goldner Harary which has 11 vertices and 27 edges

**Case: 1**

$$V(G) = \{v_1, v_2, \dots, v_{11}\}, E(G) = \{e_1, e_2, \dots, e_{27}\}$$

$$|V(G)| = 11 \quad |E(G)| = 27$$

Let  $G^1$  be a graph obtained by fusion of two vertices  $v_7$  and  $v_3$

Let the fusion vertex be  $u$ .

$$\text{Then } |V(G^1)| = 10 \quad |E(G^1)| = 24$$

Define a labeling  $f: V(G^1) \rightarrow \{1, 2, \dots, 11\}$

$$\text{such that } f(v_i) = i, \quad i = \{1, 2, 3, 5, 9, 11\}$$

$$f(v_i) = 3i + 1, \quad i = \{4, 6, 8, 10\}$$

$$f(v_3) = f(u), \quad i = 3$$

The relative prime of adjacent vertices have to be verify

We look at the following types of edges:

$$\text{Gcd}\{f(v_i), f(v_{i+1})\} = 1 \quad \text{for } i = 1, 2, 4, 5, 6$$

$$\text{Gcd}\{f(v_8), f(v_2)\} = 1$$

$$\text{Gcd}\{f(v_1), f(v_4)\} = 1$$

$$\text{Gcd}\{f(v_2), f(v_9)\} = 1$$

$$\text{Gcd}\{f(v_9), f(v_4)\} = 1$$

$$\text{Gcd}\{f(v_4), f(v_{10})\} = 1$$

$$\text{Gcd}\{f(v_{10}), f(v_6)\} = 1$$

$$\text{Gcd}\{f(v_1), f(v_{10})\} = 1$$

$$\text{Gcd}\{f(v_1), f(v_{11})\} = 1$$

$$\text{Gcd}\{f(v_1), f(v_8)\} = 1$$

$$\text{Gcd}\{f(v_4), f(v_6)\} = 1$$

$$\text{Gcd}\{f(v_2), f(v_4)\} = 1$$

$$\text{Gcd}\{f(v_{11}), f(v_6)\} = 1$$

$$\text{Gcd}\{f(u), f(v_5)\} = 1$$

$$\text{Gcd}\{f(v_1), f(v_9)\} = 1$$

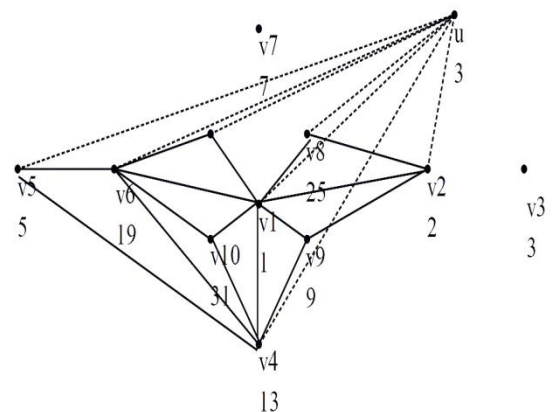
$$\text{Gcd}\{f(u), f(v_4)\} = 1$$

$$\text{Gcd}\{f(u), f(v_6)\} = 1$$

$$\text{Gcd}\{f(u), f(v_8)\} = 1$$

$$\text{Gcd}\{f(u), f(v_{11})\} = 1$$

$$\text{Gcd}\{f(u), f(v_2)\} = 1$$



Thus  $f$  is a prime labeling and consequently  $G^1$  is a prime graph.

Therefore, fusion of vertices  $v_7$  and  $v_3$  in a Goldner Harary graph admits prime labeling.

**Case: 2**

$$V(G) = \{v_1, v_2, \dots, v_{11}\}, E(G) = \{e_1, e_2, \dots, e_{27}\}$$

$$|V(G)| = 11 \quad |E(G)| = 27$$

Let  $G^1$  be a graph obtained by fusion of two vertices  $v_7$  and  $v_5$

Let the fusion vertex be  $u$ .

$$\text{Then } |V(G^2)| = 10 \quad |E(G^2)| = 24$$

Define a labeling  $f: V(G^2) \rightarrow \{1, 2, \dots, 10\}$

$$\text{such that } f(v_i) = i, \quad i = \{1, 2, 3, 7, 9, 11\}$$

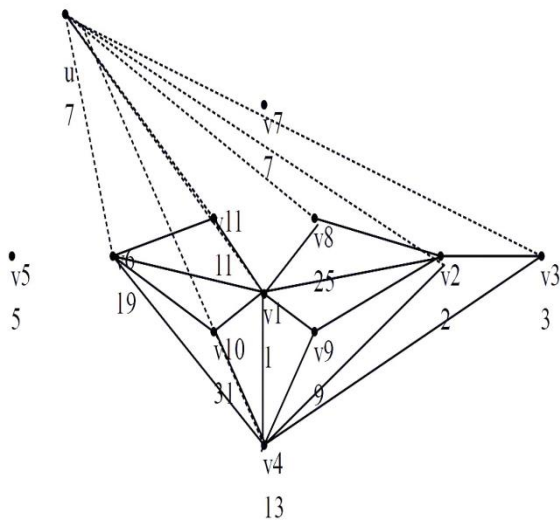
$$f(v_i) = 3i + 1, \quad i = \{4, 6, 8, 10\}$$

$$f(v_7) = f(u), \quad i = 7$$

The relative prime of adjacent vertices have to be verify

We look at the following types of edges:

- $\text{Gcd}\{f(v_i), f(v_{i+1})\} = 1$  for  $i=1,2,3,4,6,$
- $\text{Gcd}\{f(v_8), f(v_2)\} = 1$
- $\text{Gcd}\{f(v_1), f(v_4)\} = 1$
- $\text{Gcd}\{f(v_2), f(v_9)\} = 1$
- $\text{Gcd}\{f(v_9), f(v_4)\} = 1$
- $\text{Gcd}\{f(v_4), f(v_{10})\} = 1$
- $\text{Gcd}\{f(v_{10}), f(v_6)\} = 1$
- $\text{Gcd}\{f(v_1), f(v_{10})\} = 1$
- $\text{Gcd}\{f(v_1), f(v_{11})\} = 1$
- $\text{Gcd}\{f(v_1), f(v_8)\} = 1$
- $\text{Gcd}\{f(u), f(v_4)\} = 1$
- $\text{Gcd}\{f(v_4), f(v_6)\} = 1$
- $\text{Gcd}\{f(v_2), f(v_4)\} = 1$
- $\text{Gcd}\{f(v_{11}), f(v_6)\} = 1$
- $\text{Gcd}\{f(u), f(v_6)\} = 1$
- $\text{Gcd}\{f(v_1), f(v_9)\} = 1$
- $\text{Gcd}\{f(u), f(v_{11})\} = 1$
- $\text{Gcd}\{f(u), f(v_8)\} = 1$
- $\text{Gcd}\{f(u), f(v_2)\} = 1$
- $\text{Gcd}\{f(u), f(v_3)\} = 1$



Thus  $f$  is a prime labeling and consequently  $G^2$  is a prime graph.

Therefore, fusion of vertices  $v_7$  and  $v_5$  in a Goldner Harary graph admits prime labeling.

**Case: 3**

$$V(G) = \{v_1, v_2, \dots, v_{11}\}, E(G) = \{e_1, e_2, \dots, e_{27}\}$$

$$|V(G)| = 11 \quad |E(G)| = 27$$

Let  $G^3$  be a graph obtained by fusion of two vertices  $v_4$  and  $v_5$

Let the fusion vertex be  $u$ .

$$\text{Then } |V(G^3)| = 10 \quad |E(G^3)| = 24$$

Define a labeling  $f: V(G^3) \rightarrow \{1, 2, \dots, 10\}$

$$\text{such that } f(v_i) = i, \quad i = \{1, 2, 3, 7, 9, 11\}$$

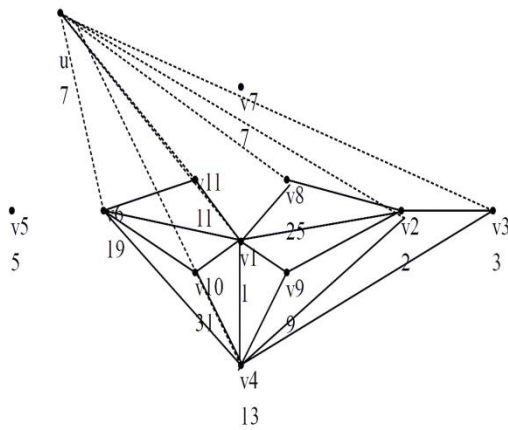
$$f(v_i) = 3i + 1, \quad i = \{4, 6, 8, 10\}$$

$$f(v_4) = f(u), \quad i = 4$$

The relative prime of adjacent vertices have to be verify

We look at the following types of edges:

- $\text{Gcd}\{f(v_i), f(v_{i+1})\} = 1$  for  $i=1,2,3,6,7$
- $\text{Gcd}\{f(v_8), f(v_2)\} = 1$
- $\text{Gcd}\{f(v_1), f(v_4)\} = 1$
- $\text{Gcd}\{f(v_2), f(v_9)\} = 1$
- $\text{Gcd}\{f(v_9), f(v_4)\} = 1$
- $\text{Gcd}\{f(v_4), f(v_{10})\} = 1$
- $\text{Gcd}\{f(v_{10}), f(v_6)\} = 1$
- $\text{Gcd}\{f(v_1), f(v_{10})\} = 1$
- $\text{Gcd}\{f(v_1), f(v_{11})\} = 1$
- $\text{Gcd}\{f(v_1), f(v_8)\} = 1$
- $\text{Gcd}\{f(v_7), f(v_3)\} = 1$
- $\text{Gcd}\{f(v_4), f(v_6)\} = 1$
- $\text{Gcd}\{f(v_2), f(v_4)\} = 1$
- $\text{Gcd}\{f(v_{11}), f(v_6)\} = 1$
- $\text{Gcd}\{f(v_7), f(v_1)\} = 1$
- $\text{Gcd}\{f(v_1), f(v_9)\} = 1$
- $\text{Gcd}\{f(v_7), f(v_4)\} = 1$
- $\text{Gcd}\{f(v_7), f(v_6)\} = 1$
- $\text{Gcd}\{f(v_7), f(v_{11})\} = 1$
- $\text{Gcd}\{f(u), f(v_7)\} = 1$
- $\text{Gcd}\{f(u), f(v_6)\} = 1$
- $\text{Gcd}\{f(u), f(v_{10})\} = 1$
- $\text{Gcd}\{f(u), f(v_9)\} = 1$
- $\text{Gcd}\{f(u), f(v_2)\} = 1$
- $\text{Gcd}\{f(u), f(v_3)\} = 1$



Thus  $f$  is a prime labeling and consequently  $G^3$  is a prime graph.

Therefore, fusion of vertices  $v_5$  and  $v_4$  in a Goldner Harary graph.

**Conclusion**

In this Paper, we discussed Goldner Harary graph admits Prime Labeling.

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