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Research Article

Bianchi type- V Cosmological modelsin the presence of a bulk viscous fluid based on Lyra geometry in F(R, T) theory of gravity

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Abstract

In this paper an attempt is made to investigate the exact solution of Bianchi type- V Cosmological models in the framework of f(R,T) specifically considering the form F(R,T)=R+2f(T) and f(T) = λ T where λ is a constant . This modified theory of gravity introduces a coupling between the Ricci scalar R and the trace of the energy-momentum tensor T, potentially offering new insights into the dynamics of the early universe. We derive the field equations for the Bianchi Type V metric and explore exact solutions under specific assumptions. The physical and geometrical properties of the models, including the behavior of the scale factors, Hubble parameter, deceleration parameter, and energy conditions, are analyzed. We find that the models exhibit a variety of cosmological behaviors, including inflationary and decelerating phases, depending on the choice of initial conditions and model parameters. Our results demonstrate the potential of F(R,T) gravity to provide alternative explanations for the observed accelerated expansion of the universe. The physical and geometrical behaviors of such models have been discussed

Key Words: *Bianchi type, Deceleration, State finder and Jurk Parameter, f(R,T) gravity.*

Introduction

The standard model of cosmology, based on General Relativity (GR), has been remarkably successful in explaining a wide range of cosmological observations. However, recent astronomical observations, such as the accelerated expansion of the universe and the observed flatness of the universe, have posed challenges to the standard model. Modified theories of gravity, which modify the Einstein-Hilbert action, have emerged as potential alternatives to address these issues.

One such modified theory is F(R,T) gravity, where the gravitational Lagrangian is an arbitrary function of the Ricci scalar R and the trace of the energy-momentum tensor T. This theory offers a natural way to incorporate the effects of matter and energy on the geometry of spacetime. In this paper, we focus on the specific form F(R,T)=R+2T, which has been extensively studied in the literature.

Bianchi Type V models are a class of anisotropic cosmological models that allow for spatial curvature and inhomogeneities. They provide a more realistic description of the early universe compared to isotropic models like the Friedmann-Lemaître-Robertson-Walker (FLRW) models. By studying Bianchi Type V models in F(R,T) gravity, we can explore the impact of modified gravity on the early universe dynamics and potentially gain insights into the origin of cosmic structures.

Objectives

To study Bianchi type -V Cosmological models in F(R,T) theory of gravity.

To find the exact solutions of corresponding field equations consistent with the resent observation related to the expansion of the universe.

To study the various physical and kinematical parameters and their properties of corresponding anisotropic cosmological model.

Methodology

F(R, T) Gravity

The action for F(R,T) gravity is given by

$$S = \int \sqrt{-g}\left[\frac{1}{16\pi G}F(R,T) + L_m\right]d^4x \quad (1)$$

Where R is Ricci Scalar

T is Trace of Energy Momentum Tensor

L_m is matter Lagrangian

Field Equation For F(R,T) Gravity

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} - (\nabla_i\nabla_j - g_{ij})f_R(R,T) = 8\pi G T_{ij} + f_T(R,T)(T_{ij} + p g_{ij}) \quad (2)$$

Where $f_R(R,T) = \frac{\partial f(R,T)}{\partial R}$

$f_T(R,T) = \frac{\partial f(R,T)}{\partial T}$

$$T_{ij} = \frac{-2\delta(L_m\sqrt{-g})}{\sqrt{-g}\,\delta g^{ij}} \quad (3)$$

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T) \end{cases} \quad (4)$$

We consider the line element of Bianchi type V space time and it is given by

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2mx}dy^2 + C^2e^{-2mx}dz^2 \quad (5)$$

Where A, B and C are metric coefficients and functions of cosmic time t and m is a arbitray constant

Solution for f(R, T) = R + 2λT where λ is a constant

The field equation takes the form

$$G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij} = (8\pi + 2\lambda)T_{ij} + \lambda(2p + T)g_{ij} \quad (a)$$

This forms looks like EFEs in GR and λ (2p +T) is replaced by Cosmological Constant ∧

i.e. $\lambda(2p + T) = \Lambda$, $T = \rho - 3p$ (perfect fluid case) (b)

$\Lambda = \lambda(\rho - p)$

Thus we rewrite Equation as

$$R_{ij} - \frac{1}{2}Rg_{ij} = (8\pi + 2\lambda)T_{ij} + \Lambda g_{ij} \quad (c)$$

Now we obtain a set of differential equations

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} = (8\pi + 2\lambda)p - \Lambda \quad (6)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{B}\dot{A}}{BA} - \frac{m^2}{A^2} = (8\pi + 2\lambda)p - \Lambda \quad (7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{m^2}{A^2} = (8\pi + 2\lambda)p - \Lambda \quad (8)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{3m^2}{A^2} = -\Lambda - (8\pi + 2\lambda)p \quad (9)$$

$$\frac{\dot{B}}{B} + \frac{\dot{C}}{C} - 2\frac{\dot{A}}{A} = 0 \quad (10)$$

Intregating We get

$A^2 = k_1$ BC

Where k_1 is an integrating constant and without loss of generality, the constant of integration k_1 can be chosen as unity

$$A^2 = BC \quad (11)$$

In the field Equations 15–19, we found that there are five equations involving seven unknowns. As the field equations are highly non-linear differential equations, we need some other condition to complete the field equations such as.

1) we consider the shear scalar (σ) is proportional to the expansion scalar (θ) [Collins et al. (Collins et al., 1980)]

$$B = C^n \quad (12)$$

Where n is a non zero constant.

2) Let us consider the combined effect of the proper pressure and the bulk viscous pressure, for a barotropic fluid can be expressed as follows

$$\bar{p} = p - 3\xi H = \varepsilon\rho;\ p = \varepsilon_0\rho \quad (13)$$

Such that $\varepsilon = \varepsilon_0 - \eta$ $(0 \le \varepsilon_0 \le 1)$ and ϵ, ε_0, and η are constant. The symbols ξ and p are respectively known as the coefficient of bulk viscosity and proper pressure of the model.

Let us consider a time dependent displacement field scale factor [Pradhan et al. (Pradhan et al., 2006)] as given by

$$a(t) = \alpha e^{t\alpha_1} \quad (14)$$

Where α and α_1 are constants. From Eqs 23, 24, 26, we get the metric potentials of the model which are

A $=\alpha e^{t\alpha_1}$, B = $(\alpha e^{t\alpha_1})^{\frac{2n}{n+1}}$, C = $(\alpha e^{t\alpha_1})^{\frac{2}{n+1}}$(15)

Then Eq. 1 reduces to

ds^2 = - dt^2 + $(\alpha e^{t\alpha_1})^2 dx^2$ $+e^{-2mx}\left[(\alpha e^{t\alpha_1})^{\frac{4n}{n+1}}dy^2+(\alpha e^{t\alpha_1})^{\frac{4}{n+1}}dz^2\right]$(16)

Physical Properties of the Model In F(R, T) Gravity

The Physical Parameters of the model are obtained as follows:

Spatial Volume:

V=a^3(t)= $(\alpha e^{t\alpha_1})^3$(17)

Hubble's Parameter:

H = $\alpha\alpha_1$(18)

the expansion scalar $\theta = 3\alpha\alpha_1$(19)

shear scalar $\sigma^2 = (\frac{n-1}{n+1})^2(\alpha\alpha_1)^2$(20)

Anisotropy parameter:

$A_m = \frac{2}{3}(\frac{n-1}{n+1})^2$(21)

The deceleration parameter: q = -1 (22)

Adding Eqs6–8 and applying in Eq. 9, we have the energy density given by

$\rho = \frac{1}{2\epsilon-1}\left[-6(\alpha\alpha_1)^2 + \frac{4m^2}{(\alpha e^{t\alpha_1})^2}\right]$(23)

Also the total pressure, proper pressure, the co-efficient of bulk viscosity and the Trace are given by

$\bar{p} = \frac{\epsilon}{2\epsilon-1}\left[-6(\alpha\alpha_1)^2 + \frac{4m^2}{(\alpha e^{t\alpha_1})^2}\right]$(24)

P = $\frac{\epsilon_0}{2\epsilon-1}\left[-6(\alpha\alpha_1)^2 + \frac{4m^2}{(\alpha e^{t\alpha_1})^2}\right]$(25)

$\xi = \frac{\epsilon_0-\varepsilon}{3(2\epsilon-1)}\left[-6(\alpha\alpha_1)^2 + \frac{4m^2}{(\alpha e^{t\alpha_1})^2}\right]$(26)

T = $\frac{1-3\epsilon}{2\epsilon-1}\left[-6(\alpha\alpha_1)^2 + \frac{4m^2}{(\alpha e^{t\alpha_1})^2}\right]$(27)

From Esq. 16, 19, we obtain the statefinder parameters, which is defined as r=$\frac{\ddot{a}}{aH^3}$ and s=$\frac{r-1}{3(q-\frac{1}{2})}$, exactly gives the value 1 and 0 respectively.

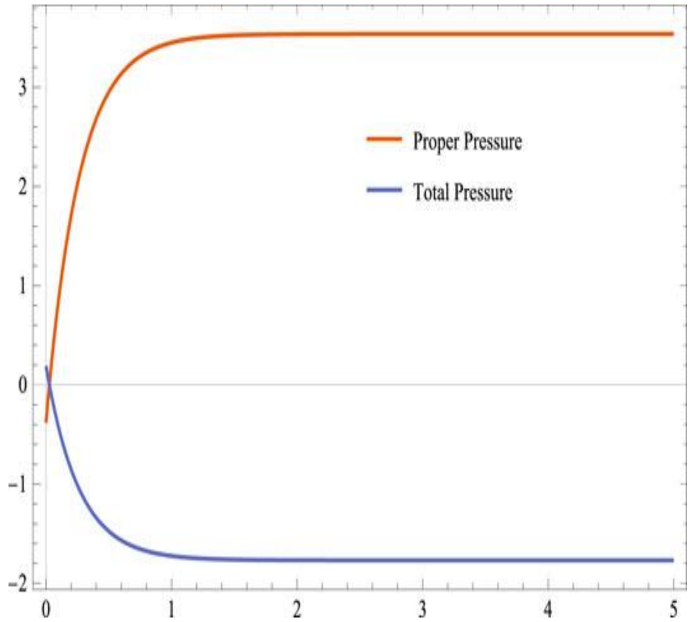

Figure: Variation of total pressure (p)($\bar{p}$) and proper pressure (*p*) *vs*. time (*t*).

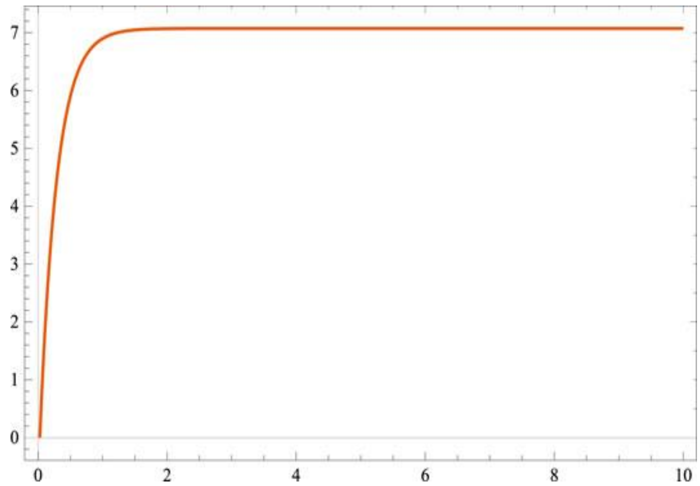

Figure: Variation of density (*ρ*) *vs*. time (*t*).

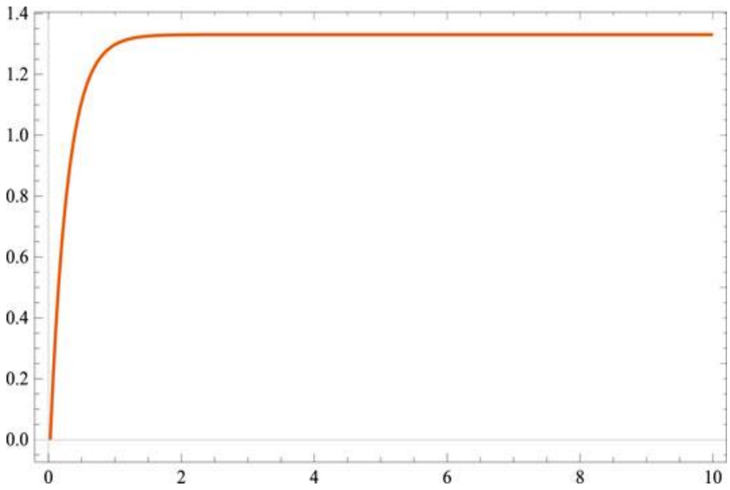

Figure: Variation of coefficient bulk viscosity (ξ) *vs*. time (t).

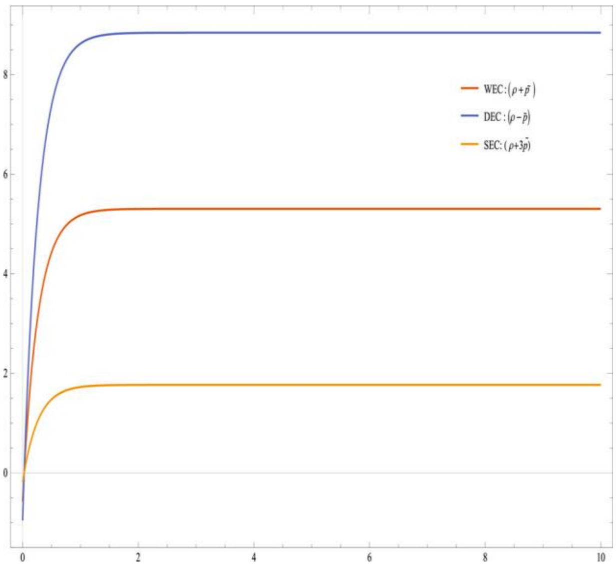

Figure: Variation of strong energy condition *vs*. time (t).

Conclusion

In this study, a completely spatially homogeneous and anisotropic Bianchi type-V cosmological model has been discussed in the presence of a bulk viscous fluid based on Lyra geometry, with an exponential form of scale factor. We employed a barotropic equation of state for pressure and energy density to determine the nature and deterministic solution of the highly non linear differential equation. Furthermore, we assumed that bulk viscous pressure is proportional to energy density [Naidu et al. (Naidu etal.,2013)].As per recent observation aldatain combination with Baryonic Acoustic Oscillations (BAO), Cosmic Microwave Background (CMB), from Type Ia Supernova (SN Ia), the model found in this paper is in conformative. The model 28) found here is shearing, expanding, and anisotropic which is similar to [Zia (Zia et al., 2018), Tiwari (Tiwari et al., 2020), and Desikan (Desikan, 2020)]. At t = 0, we found that the model has no singularity. Subsequently, we can see from Eqs18, 19 that the Hubble's parameter (H) and the expansion scalar remain constant throughout the expansion, implying that model 28) represents a uniform expansion. It is evident from Eq. 17 that the volume (V) of the universe rises with cosmic time (t), and that as V approaches infinity, for t →∞. We observed from Eq. 26 that the bulk viscosity coefficient increases with time and approaches infinity as t approaches infinity. Themodel's energy density, total pressure, coefficient of bulk viscosity, and displacement vector all rise positively, but they all yield a constant value for t tending to infinity (Figures 1–3). The model predicts an accelerating phase of the universe for q = − 1, which is given by Eq. 22. In the current model of the universe, there is a dark energy due to negative pressure in the presence of bulk viscous fluid based on Lyra geometry with f (R,T)gravity,as shown in Figure1.All the three energy conditions are satisfied as in Figure 4. The Trace is always positive throughout the cosmic time t, and fort → ∞, it offers a constant value, as shown in Eqs27 . Furthermore, r and s tend to 1 and 0 respectively, indicating that the current universe model approaches the ΛCDM model. There have been many works done by researchers in the area of Lyra geometry, but Lyra geometry with f(R,T) gravity is a very new concept and there is scope for the continuation of work.

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