LouisSaveinDupuisJournalofMultidisciplinaryResearch2024,3
281-291: Doi.: 10.21839/lsdjmr.2024.v3.193

Research Article

SOME RESULTS ON LORENTZIAN α-SASAKIAN MANIFOLDS

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Abstract

The purpose of this work is to introduce Lorentzian α-Sasakian manifolds to the concept of an extended w_2-curvature tensor. This study's findings include the demonstration of an extended Lorentzian α-Sasakian manifold that satisfies certain requirements for the w_2-curvature tensor. First we demonstrated that, it is local isometric to the hyperbolic space because a Lorentzian α-Sasakian manifold satisfying $w_2 = 0$ is a space with constant curvature -1. Further, we proved that, Einstein manifolds are w_2-semisymmetric Lorentzian α-Sasakian M manifolds. Additionally we validated that, the hyperbolic space is locally isometric to a w_2-semisymmetric Lorentzian α-Sasakian manifold, that is a space with constant curvature of -1. Additionally, we gathered data on an Einstein manifold is a M that is a $B(X,Y) \cdot w_2 = 0$ satisfying Lorentzian α-Sasakian manifold, a M Lorentzian α-Sasakianmanifold that satisfies the $C(X,Y) \cdot w_2 = 0$ condition is an Einstein manifold and Einstein manifolds are Lorentzian α-Sasakian manifolds that meet the equation$P(X,Y) \cdot w_2 = 0$.

Key Words*: Lorentzian α-Sasakian, C-Bouchner curvature tensor, Weyl-conformal curvature tensor, Weyl-projective curvature tensor and Einstein manifold.*

MSC Subject Classification (2000): 53D15, 53C21, 53C25, 53C40.

Introduction

Pokhariyal and Mishra [9] introduced the w_2-curvature tensor, a novel type of curvature tensor, in a Riemannian manifold in the year 1770 and researched its features. Pokhariyal [10] has also investigated some of the characteristics of this curvature tensor in a Sasakian manifold. In P-Sasakian, Kenmotsu and Lorentzian para-Sasakian manifolds, w_2-curvature tensor has been explored by Matsumoto, Ianu and Mihai [12], Ahmet Yildiz and U.C. De [21] and Venkatesha, C.S. Bagewadi et al [20], respectively. S. Tanno classified associated almost contact metric manifolds with the largest automorphism grou in [17]. The sectional curvature of a plane section containing such a manifold is a constant, let's say c He showed how they can be classified into three groups:

Riemannian manifolds with homogeneous normal contact and $c > 0$;
In the condition that $c = 0$, global Riemannian products of a line or circle with a Kaehler Manifold with constant holomorphic sectional curvature;
If $c > 0$, a warped product space calculated as $R_f \times C$. The class (1) manifolds are distinguished by admitting a Sasakian structure, as is well known.

A class of nearly Hermitian manifolds [6],w_4, that is strongly connected to locally conformal Kaehler manifolds [3] appears in the Gray-Hervella classification of almost Hermitian manifolds. A trans-Sasakian structure [16] is a nearly contact metric structure on a manifold M if the product manifold $M \times R$ belongs to the class w_4. The class of the trans-Sasakian structures of (α, β)

coincides with the class $C_6 \oplus C_5$ ([14], [15]). In fact, the local nature of the two subclasses of trans-Sasakian structures, namely C_5 and C_6 structures, is fully defined in [15]. We point out that the cosymplectic [1], β-Kenmotsu [8] and α-Sasakian [8], respectively, are trans-Sasakian structures of type

(0,0), $(0,\beta)$ and $(\alpha,0)$. It is established in [18] that trans-Sasakian structures are generalized quasi-Sasakian structures. As a result, a wide variety of generalized quasi-Sasakian structures are also provided by trans-Sasakian structures.

If $M \times R$ belongs to the class w_4 [6] and J is the almost complex structure on $M \times R$ defined by

$(M \times R, J, G)$ [16], then the almost contact metric structure (ϕ, ξ, η, g) on M

$$J\left(X, \frac{fd}{dt}\right) = \left(\phi X - f, \frac{\eta(X)d}{dt}\right). \tag{1.1}$$

The product metric on $M \times R$ is G for all vector fields X on M and smooth functions f on$M \times R$.

This might be stated using the condition [2]

$$(\nabla_X \phi)Y = \alpha(g(X,Y) - \eta(Y)X) + \beta(g(\phi X, Y) - \eta(Y)\phi X)), \tag{1.2}$$

For certain smooth functions (α,β) and (β,α) on M, we say that the trans-Sasakian structure is of type (α,β). A trans-Sasakian structure of type (α,β) is α-Sasakian if $\beta = 0$ and α is a non-zero Constant [7]. The α-Sasakian manifold is a Sasakian manifold if $\alpha = 1$.

Preliminaries

The term"differentiable manifold of dimension n". If it admits a Lorentzian α-Sasakian manifold, a contravariant vector field namedξ, a (1,1)-tensor field named ϕ and a Lorentzian metric g covariant vector field η satisfy ([22], [13])

$$\eta(\xi) = -1, \tag{2.1}$$

$$\phi^{\ 2} = X + \eta(X)\xi, \tag{2.2}$$

$$g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y), \tag{2.3}$$

$$g(X,\xi) = \eta(X), \tag{2.4}$$

$$\phi\xi = 0, \qquad \eta(\phi X) = 0, \tag{2.5}$$

For every $X, Y \in TM$.

A Lorentzian α-Sasakian manifold M [4] also satisfies the condition

$$\nabla_{\mathrm{x}}\xi = -\alpha\phi X, \tag{2. 6}$$

$$(\nabla_{\mathrm{x}}\eta)Y = -\alpha g(\phi X, Y), \tag{2. 7}$$

Where the covariant differentiation operator with respect to the Lorentzian metric g is denoted by ∇.

The following relations hold on Lorentzian α-Sasakian manifold M as well:

$$R(X,Y)Z = \alpha^2\{g(Y,Z)X - g(X,Z)Y\}, \tag{2.8}$$

$$R(X,Y)\xi = \alpha^2\{\eta(Y)X - \eta(X)Y\}, \tag{2.9}$$

$$R(\xi,X)Y = \alpha^2\{g(X,Y)\xi - \eta(Y)X\}, \tag{2.10}$$

$$R(\xi,X)\xi = \alpha^2\{\eta(X)\xi + X\}, \tag{2.11}$$

$$S(X,\xi) = (n-1)\alpha^2\eta(X), \tag{2.12}$$

$$Q\xi = (n-1)\alpha^2\xi, \tag{2.13}$$

$$S(\phi X, \phi Y) = S(X,Y) + (n-1)\alpha^2\eta(X)\eta(Y), \tag{2.14}$$

Where S is the Ricci tensor and Q is the Ricci operator provided by, for any vector fields X, Y and Z.

$$S(X,Y) = \alpha g(X,Y), \tag{2.15}$$

Any vector field X, any vector field Y and α is a function on M.

The definition of the curvature tensor w_2 by Pokhariyal and Mishra [9] is given in the

$$w_2(X,Y,U,V) = R(X,Y,U,V) + \frac{1}{(n-1)}[g(X,U)S(Y,V) - g(Y,U)S(X,V)], \tag{2.16}$$

Where S is a Ricci tensor with the form(0,2).

Assume an Lorentzian α-Sasakian manifold satisfying$w_2 = 0$; in this case, (2.16) becomes true.

$$R(X,Y,U,V) = \frac{1}{(n-1)}[g(Y,U)S(X,V) - g(X,U)S(Y,V)]. \quad (2.17)$$

Using (2.17)'s $X = U = \xi$ and (2.9), (2.12), we have

$$S(Y,V) = \alpha^2(n-1)g(Y,V). \quad (2.18)$$

M Is therefore an Einstein manifold.

Once more inserting (2.18) into (2.17), we obtain

$$R(X,Y,U,V) = \alpha^2[g(Y,U)g(X,V) - g(X,U)g(Y,V)]. \quad (2.19)$$

Corollary : It is local isometric to the hyperbolic space because a Lorentzian α-Sasakian manifold satisfying $w_2 = 0$ is a space with constant curvature -1

Definition: If a w_2-semisymmetric Lorentzian α-Sasakian manifold satisfies

$$R(X,Y)\cdot w_2 = 0, \quad (2.20)$$

Where the derivation of the tensor algebra for the tangent vectors X and Y at each point on the manifold Is to be thought of as$R(X,Y)$.

The condition is easily demonstrated for the w_2-curvature tensor in a Lorentzian α-Sasakian manifold.

$$\eta(w_2(X,Y)Z) = 0. \quad (2.21)$$

Theorem 2.1. An Einstein manifolds are w_2-semisymmetric Lorentzian α-Sasakian M manifolds.

Proof: As a result of$R(X,Y)\cdot w_2 = 0$, we have

$$R(X,Y)w_2(U,V)Z - w_2(R(X,Y)U,V)Z - w_2(U,R(X,Y)V)Z - w_2(U,V)R(X,Y)Z = 0. \quad (2.22)$$

We obtain by entering $X = \xi$ in (2.22), then taking the inner product withξ

$$g(R(\xi,Y)w_2(U,V)Z,\xi) - g(w_2(R(\xi,Y)U,V)Z,\xi)$$
$$-g(w_2(U,R(\xi,Y)V)Z,\xi) - g(w_2(U,V)R(\xi,Y)Z,\xi) = 0. \quad (2.23)$$

In (2.23), we obtain using (2.10).

$$0 = -\alpha^2 g(Y,w_2(U,V)Z) - \alpha^2\eta(w_2(U,V)Z)\eta(Y) - \alpha^2 g(Y,U)\eta(w_2(\xi,V)Z)$$
$$+\alpha^2\eta(U)\eta(w_2(Y,V)Z) - \alpha^2 g(Y,V)\eta(w_2(U,\xi)Z) + \alpha^2\eta(V)\eta(w_2(U,Y)Z)$$
$$-\alpha^2 g(Y,Z)\eta(w_2(U,V)\xi) + \alpha^2\eta(Z)\eta(w_2(U,V)Y). \quad (2.24)$$

When we put (2.21) in (2.24), we get

$$\alpha^2\, w_2(U,V,Z,Y) = 0. \quad (2.25)$$

Considering [(2.16) and (2.25)], it follows that

$$R(U,V,Z,Y) = \frac{1}{(n-1)}[g(V,Z)S(U,Y) - g(U,Z)S(V,Y)]. \quad (2.26)$$

Using a contract (2.26), we have

$$S(V,Z) = \alpha^2(n-1)g(V,Z). \quad (2.27)$$

With reference to (2.17) and (2.27) once more, we obtain

$$R(U,V,Z,Y) = \alpha^2[g(V,Z)g(U,Y) - g(U,Z)g(V,Y)].$$

(2.26)

Corollary 2.2. The hyperbolic space is locally isometric to a w_2-semisymmetric Lorentzian α-Sasakian manifold, that is a space with constant curvature of -1.

1. Engaging Lorentzian α-Sasakian Manifolds with $B(X,Y)\cdot w_2 = 0$

As stated in the definition of the C-Bouchner curvature tensor B [11]

$$\begin{aligned} B(X,Y)Z &= R(X,Y)Z + \frac{1}{(n+3)}[S(X,Z)Y - S(Y,Z)X + g(X,Z)QY - g(Y,Z)QX + S(\phi X,Z)\phi Y \\ &-S(\phi Y,Z)\phi X + g(\phi X,Z)Q\phi Y - g(\phi Y,Z)Q\phi X + 2S(\phi X,Z)\phi Y + 2g(\phi X,Y)Q\phi Z \\ &-S(X,Z)\eta(Y)\xi + S(Y,Z)\eta(X)\xi - \eta(X)\eta(Z)QY + \eta(Y)\eta(Z)QX] \\ &-\frac{(p+n-1)}{(n+3)}[g(\phi X,Z)\phi Y - g(\phi Y,Z)\phi X + 2g(\phi X,Y)\phi Z] - \frac{(p-4)}{(n+3)}[g(X,Z)Y - g(Y,Z)X] \\ &+\frac{p}{(n+3)}[g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi + \eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X]. \end{aligned}$$

(3.1)

When $X = \xi$ in (3.1) is reduced using (2.10) and (2.12),

$$B(\xi,Y)Z = K_1 g(Y,Z)\xi + K_2\eta(Z)Y + K_3 S(Y,Z)\xi,$$

(3.2)

where $K_1 = \left[\alpha^2 - \alpha^2\frac{(n-1)}{(n+3)} + \frac{(p-4)}{(n+3)} + \frac{(p)}{(n+3)}\right]$, $K_2 = \left[-\alpha^2 + 3\alpha^2\frac{(n-1)}{(n+3)} - \frac{(p-4)}{(n+3)} - \frac{(p)}{(n+3)}\right]$ and $K_3 = \frac{-2}{(n+3)}$.

Consider the possibility that in a Lorentzian α-Sasakian manifold

$$B(X,Y)\cdot w_2 = 0.$$

(3.3)

According to this condition,

$$B(X,Y)w_2(U,V)Z - w_2(B(X,Y)U,V)Z - w_2(U,B(X,Y)V)Z - w_2(U,V)B(X,Y)Z = 0. \tag{3.4}$$

When we enter $X = \xi$ in (3.4) and take the inner product with ξ, we get

$$\begin{aligned} &g(B(\xi,Y)w_2(U,V)Z,\xi) - g(w_2(B(\xi,Y)U,V)Z,\xi) \\ &\quad -g(w_2(U,B(\xi,Y)V)Z,\xi) - g(w_2(U,V)B(\xi,Y)Z,\xi) = 0. \end{aligned} \tag{3.5}$$

Utilization (2.21), (3.2) in (3.5), in our case,

$$\begin{aligned} 0 = &-K_1 g(Y,w_2(U,V)Z) - K_3 S(Y,w_2(U,V)Z) - K_1 g(Y,U)\eta(w_2(\xi,V)Z) \\ &-K_3 S(Y,U)\eta(w_2(\xi,V)Z) - K_1 g(Y,V)\eta(w_2(U,\xi)Z) - K_3 S(Y,V)\eta(w_2(U,\xi)Z) \\ &-K_1 g(Y,Z)\eta(w_2(U,V)\xi) - K_3 S(Y,Z)\eta(w_2(U,V)\xi). \end{aligned} \tag{3.6}$$

When we put (2.21) in (3.6), we obtain

$$0 = K_1 g(Y,w_2(U,V)Z) + K_3 S(Y,w_2(U,V)Z). \tag{3.7}$$

Utilizing [(2.16) and (2.10)] and $U = Z = \xi$, we have

$$S(V,QY) = \left[\frac{K_1(n-1)\alpha^2}{K_3}\right] g(V,Y) + \left[(n-1)\alpha^2 - \frac{K_1}{K_3}\right] S(V,Y). \tag{3.8}$$

The result of this is

$$QY = \alpha^2(n-1)Y. \tag{3.9}$$

This gives us

$$S(Y,V) = \alpha^2(n-1)g(Y,V). \tag{3.10}$$

The following can be said as a result:

Theorem 3.2. An Einstein manifold is a M that is a $B(X,Y)\cdot w_2 = 0$ satisfying Lorentzian α-Sasakian manifold.

2. Engaging Lorentzian α-Sasakian Manifolds with $\mathbf{C}(X,Y)\cdot w_2 = 0$

As stated in the definition of the Weyl-conformal curvature tensor [5], C

$$C(X,Y)Z = R(X,Y)Z - \frac{1}{(n-2)}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY] + \frac{r}{(n-1)(n-2)}[g(Y,Z)X - g(X,Z)Y]. \quad (4.1)$$

When $X = \xi$ in (4.1) is reduced using (2.10) and (2.12),

$$C(\xi,Y)Z = K_1' g(Y,Z)\xi + K_2' \eta(Z)Y + K_3' S(Y,Z)\xi, \quad (4.2)$$

where $K_1' = \left[\alpha^2 - \alpha^2\frac{(n-1)}{(n-2)} + \frac{r}{(n-1)(n-2)}\right]$, $K_2' = \left[-\alpha^2 + 2\alpha^2\frac{(n-1)}{(n-2)} - \frac{r}{(n-1)(n-2)}\right]$ and $K_3' = \frac{-1}{(n-2)}$.

Suppose that in an Lorentzian α-Sasakian manifold manifold

$$C(X,Y)\cdot w_2 = 0. \quad (4.3)$$

According to this condition,

$$C(X,Y)w_2(U,V)Z - w_2(C(X,Y)U,V)Z - w_2(U,C(X,Y)V)Z - w_2(U,V)C(X,Y)Z = 0. \quad (4.4)$$

Using the formula $X = \xi$ in the reference equation (4.4) and the inner product with ξ, we get

$$g(C(\xi,Y)w_2(U,V)Z,\xi) - g(w_2(C(\xi,Y)U,V)Z,\xi)$$
$$-g(w_2(U,C(\xi,Y)V)Z,\xi) - g(w_2(U,V)C(\xi,Y)Z,\xi) = 0.$$

(4.5)

Using (2.21), (4.2) in (4.5), we obtain

$$0 = -K_1' g(Y, w_2(U,V)Z) - K_3' S(Y, w_2(U,V)Z) - K_1' g(Y,U)\eta(w_2(\xi,V)Z)$$
$$-K_3' S(Y,U)\eta(w_2(\xi,V)Z) - K_1' g(Y,V)\eta(w_2(U,\xi)Z) - K_3' S(Y,V)\eta(w_2(U,\xi)Z)$$
$$-K_1' g(Y,Z)\eta(w_2(U,V)\xi) - K_3' S(Y,Z)\eta(w_2(U,V)\xi).$$

(4.6)

By inserting (2.21) into (4.6), we obtain

$$0 = K_1' g(Y, w_2(U,V)Z) + K_3' S(Y, w_2(U,V)Z).$$

(4.7)

Taking $U = Z = \xi$ and in view of (2.16) and (2.10), we have

$$S(V, QY) = \left[\frac{K_1'(n-1)\alpha^2}{K_3'}\right] g(V,Y) + \left[(n-1)\alpha^2 - \frac{K_1'}{K_3'}\right] S(V,Y).$$

(4.8)

This shows that

$$QY = \alpha^2(n-1)Y.$$

(4.9)

That generates

$$S(Y,V) = \alpha^2(n-1)g(Y,V).$$

(4.10)

The following can be said as a result.

Theorem 4.3. A M Lorentzian α-Sasakian manifold that satisfies the $C(X,Y)\cdot w_2 = 0$ condition is an Einstein manifold.

3. Engaging Lorentzian α-Sasakian Manifolds with $P(X,Y)\cdot w_2 = 0$

The Weyl-projective curvature tensor P is defined as [19]

$$P(X,Y)Z = R(X,Y)Z - \frac{1}{(n-1)}[S(Y,Z)X - S(X,Z)Y]. \tag{5.1}$$

When $X = \xi$ in (5.1) is reduced using (2.10) and (2.12), it becomes

$$P(\xi,Y)Z = \alpha^2 g(Y,Z)\xi - \frac{1}{(n-1)}S(Y,Z)\xi. \tag{5.2}$$

Now examine the satisfying an Lorentzian α-Sasakian manifold.

$$P(X,Y)\cdot w_2 = 0. \tag{5.3}$$

This condition shows that

$$P(X,Y)w_2(U,V)Z - w_2(P(X,Y)U,V)Z - w_2(U,P(X,Y)V)Z - w_2(U,V)P(X,Y)Z = 0. \tag{5.4}$$

When $X = \xi$ is entered into (5.4) and the inner product is taken, we get

$$g(P(\xi,Y)w_2(U,V)Z,\xi) - g(w_2(P(\xi,Y)U,V)Z,\xi) \\ -g(w_2(U,P(\xi,Y)V)Z,\xi) - g(w_2(U,V)P(\xi,Y)Z,\xi) = 0. \tag{5.5}$$

We obtain using (2.21), (5.2) in (5.5)

$$0 = -\alpha^2 g(Y,w_2(U,V)Z) + \frac{1}{(n-1)}S(Y,w_2(U,V)Z) - \alpha^2 g(Y,U)\eta(w_2(\xi,V)Z) \\ +\frac{1}{(n-1)}S(Y,U)\eta(w_2(\xi,V)Z) - \alpha^2 g(Y,V)\eta(w_2(U,\xi)Z) + \frac{1}{(n-1)}S(Y,V)\eta(w_2(U,\xi)Z) \\ -\alpha^2 g(Y,Z)\eta(w_2(U,V)\xi) + \frac{1}{(n-1)}S(Y,Z)\eta(w_2(U,V)\xi). \tag{5.6}$$

When we put (2.21) in (5.6), we obtain

$$0 = -\alpha^2 g(Y,w_2(U,V)Z) + \frac{1}{(n-1)}S(Y,w_2(U,V)Z). \tag{5.7}$$

By utilizing (2.16) and (2.10) and $U = Z = \xi$, we have

$$S(V, QY) = -\alpha^4(n-1)^2 g(V,Y) + 2\alpha^2(n-1)S(V,Y).$$
(5.8)

According to this,

$$QY = \alpha^2(n-1)Y,$$
(5.9)

which results

$$S(Y,V) = \alpha^2(n-1)g(Y,V).$$
(5.10)

Consequently, we can claim that

Theorem 5.4. Einstein manifolds are Lorentzian α-Sasakian manifolds that meet the equation $P(X,Y) \cdot w_2 = 0$.

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