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Research Article

MATRICES USED IN GRAPH THEORY

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Abstract

Graph theory relies heavily on matrix representations to model and analyze relationships between entities. Matrices provide a structured framework for encoding graph data and deriving meaningful insights through mathematical operations. This paper explores various types of matrices employed in graph theory, including adjacency matrices, incidence matrices, and path matrices. Each matrix type serves distinct purposes, such as determining connectivity, path lengths, and spectral properties of graphs. The study reviews fundamental matrix operations and their applications in solving graph theoretical problems, such as finding spanning trees, calculating graph matrices, and analyzing network robustness. By presenting a comprehensive overview of matrix theory in graph analysis, this paper contributes to enhancing the theoretical foundation and practical applications of graph theory in various domains.

Keywords: Graph, Matrices, Adjacency, Incidence, Path, Cut set, Circuit

Introduction

A diagrammatic representation of a graph may give a limited usefulness. We can represent the graph by another important mathematical structure. Matrices which is very useful in computer processing. Here in this paper we represent the graph with the help of matrix having elements 0 and 1, matrices can be used as input data by computers to study graph theory. This paper is actually a simplified representation of graph in matrix.

In Graph theory, The Matrix Representation provides a concise way to represent the relationships between vertices in a graph. Two commonly used matrices for this purpose are the adjacent matrix and the Incident matrix.

The Adjacent matrix captures the connections between vertices, while the Incident matrix represents the relationship between vertices and edges. If you are referring to cut-set matrices, Path matrices and circuits in the context of graph theory, it's common to discuss cut sets and circuits rather than matrices directly. These matrix Representations serve as powerful tools for analyzing and manipulating graphs, facilitating the application of mathematical operations and algorithms in the study of complex network structures.

Definitions

Weighted graph

The weighted graph is a graph in which a non – negative real number $w(e)$ is assigned to its each edge, the number $w(e)$ is called the weighted of the edge e.

Example

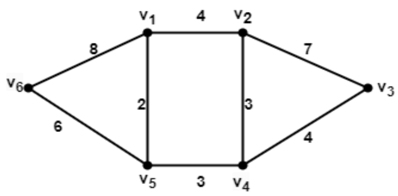


Figure 2.1, $w(v_1,v_2) = 4$

Path

A walk is called path if all its vertices are distinct.

Example

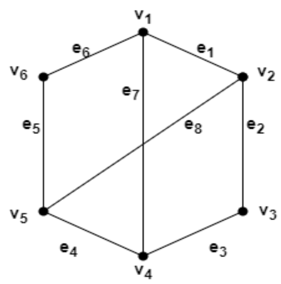


Figure 2.2 path $v_1e_1v_2e_2v_3e_3v_4e_4v_5e_5v_6$

Circuit

A closed trail is called a Circuit**.**

Example

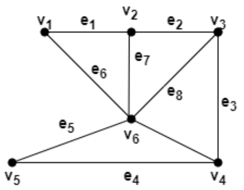


Figure 2.3Circuit**:**$v_1e_1v_2e_2v_3e_3v_4e_4v_5e_5v_6e_6v_1$

Cut set of a graph

The Cut set of a graph is a set of edges whose removal disconnects the graph or increases the number of connected components in the graph.

Example

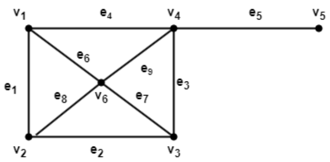


Figure 2.5Cut set = $\{e_4, e_9, e_3\}$

The matrix represents the relation between cut-set voltages and branch voltages.

The rows of a matrix represent the cut – set voltages.

The column of a matrix represents the branches of the graph.

The order of the cut – set matrix is (n-1) $\times b$.

The rank of a cut-set matrix is (n-1).

Reduced incident matrix

If one of the nodes in the given graph is considered as a reference node, then that row can be neglected by writing an incidence matrix is called a reduced incident matrix.

The order of the reduced incident matrix is (n-1)×b.

The matrix is obtained by eliminating the one of the rows (last row) of the incidence matrix.

How to find the number of trees in a graph?

The number of possible trees in a graph is given by,

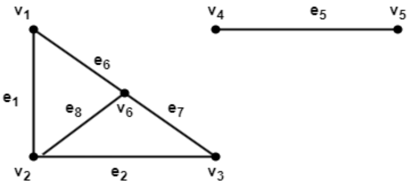
Tree = determinant [A] $[A]^T$.

[A] = reduced incidence matrix

$[A]^T$= Transpose of reduced incidence matrix.

Ring sum

Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ be two graphs. Then the ring sum of G_1 and G_2, denoted by $G_1 \oplus G_2$ is defined as the graph G such that ,



1. V(G) = V(G_1) ∪ $V(G_2)$

2. V(E) = E(G_1) ∪ $E(G_2) -$ E(G_1) ∩ $E(G_2)$.

Matrix Representation

Incident Matrix

An edge connected to a vertex is known as the Incident edge to that vertex .let G be a graph with n vertices and m edges, and without self-loops.

The Incident matrix A of G is an $n \times m$ matrix $A = [a_{ij}]$ whose n rows correspond to the n vertices and the m columns correspond to m edges such that,

$$a_{ij} = \begin{cases} 1, if\ j^{th}\ edge\ m_j\ incident\ on\ the\ i^{th}\ vertex, \\ 0, otherwise \end{cases}$$

It is also called vertex- edge incident matrix and is denoted by $A(G)$.

For instance:

It's often used to represent the connections between vertices and edges. In its simplest form, a 0-1 matrix can be used, where rows represent vertices and columns represent edges, with entries indicating whether a vertex is incident to an edge(1) or not (0).

Example

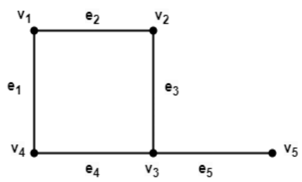


Figure 3.1

The incident matrix of figure 3.1 is,

$e_1 \quad e_2 \quad e_3 \quad e_4 \quad e_5$

$$A(G) = \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{matrix} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Applications of Incident Matrix

Graphical Representation

Incident matrices are used as a concise representation of graphs, particularly in scenarios where the focus is on edges and their connections to vertices.

Network Flow Analysis

In transportation and flow networks, incidence matrices help model the flow of goods, information, or energy between different nodes and edges.

Adjacent Matrix

An adjacent matrix is a square matrix used to represent a finite graph. When two vertices are connected by single path then they are known as adjacent vertices. If a vertex is connected to itself then the vertex is said to be adjacent to itself.

Let G be a graph with n vertices, m edges. The adjacency matrix A of G is an n$\times n$ matrix.$A = [a_{ij}]$ Whose n rows correspond to the n vertices and the m columns correspond to m edges such that,

$$a_{ij} = \begin{cases} 1, if\ (v_i, v_j) is\ an\ edge\ of\ G \\ 0, otherwise \end{cases}$$

$a_{ij} =$ Number of edges between v_i and v_j .

A value of 1 typically indicates an edge between the vertices, while a value of 0 indicates no edge. For a weighted graphs, the matrix can also store the weight of the edge between vertices.

Example for Undirected Graph

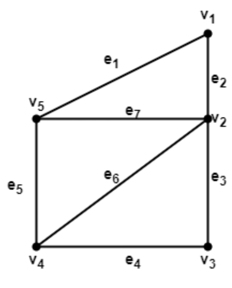


Figure 3.2.3The Adjacent matrix of figure 3.2.3 given by :

$$A(G) = \begin{matrix} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{matrix} \begin{matrix} \begin{matrix} v_1 & v_2 & v_3 & v_4 & v_5 \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

Example for weighted graph:

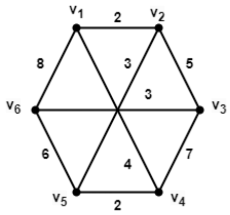


Figure 3.2.4

The adjacent matrix of figure 3.2.4 given by,

$$A(G) = \begin{matrix} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{matrix} \begin{matrix} \begin{matrix} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{matrix} \\ \begin{bmatrix} 0 & 2 & 0 & 4 & 0 & 8 \\ 2 & 0 & 5 & 0 & 3 & 0 \\ 0 & 5 & 0 & 7 & 0 & 3 \\ 4 & 0 & 7 & 0 & 2 & 0 \\ 0 & 3 & 0 & 2 & 0 & 6 \\ 8 & 0 & 3 & 0 & 6 & 0 \end{bmatrix} \end{matrix}$$

Applications of Adjacent Matrix

The adjacent matrix in graph theory has several applications, providing a compact way to represent the relationships between vertices in a graph. Some common applications include:

Graphical Representation

The adjacent matrix is a fundamental tool for representing graphs in computer science and mathematics. It efficiently captures the connections between vertices.

Circuit Matrix

Circuit is a closed walk in which no vertex or edge can appear twice. Consider a graph G = (V, E) which contain circuit. We enumerate the circuits of G: C_1, C_2,…….,C_p. there are p circuits and m edges, the circuit matrix of G is an p$\times m$ matrix C = [c_{ij}]. Circuit matrix is also called cycle matrix. The cycle matrix C of a graph G is denoted by C(G). Where,

$$c_{ij} = \begin{cases} 1, if\ i^{th}\ circuit\ includes\ j^{th}\ edge, \\ 0, othrwise \end{cases}$$

A circuit matrix is a matrix representation of a graph's circuits. Each column corresponds to an edge, and each row corresponds to a circuit in the graph. The entries are binary, indicating whether an edge is part of the corresponding circuit or not. This matrix helps analyze circuits and paths in a graph.

Example Consider the graph,

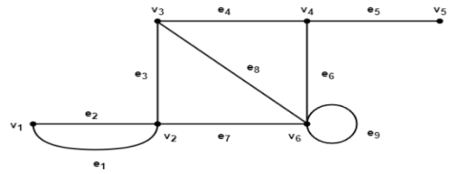


Figure 3.3.1

The graph G has five different circuits: c_1={e_1, e_2}, $c_2 = \{e_3, e_4, e_6, e_7\}$,

$$c_3 = \{e_3, e_7, e_8\},$$

$c_4 = \{e_4, e_6, e_8\}$ and $c_5 = \{e_9\}$

The circuit matrix of graph 3.3.1 given by :

C (G) =

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9
c_1	1	1	0	0	0	0	0	0	0
c_2	0	0	1	1	0	1	1	0	0
c_3	0	0	1	0	0	0	1	1	0
c_4	0	0	0	1	0	1	0	1	0
c_5	0	0	0	0	0	0	0	0	1

Applications of Circuit Matrix

Circuit Analysis

It helps in the analysis of circuits within a graph, identifying loops, and understanding the connectivity between vertices.

Graph Algorithms

It is used in algorithms that involve graph traversal, connectivity, and cycle detection. The matrix can be manipulated to find paths and cycles efficiently.

Cut-Set Matrix

Cut set is a set of edges in a graph whose removal leaves the graph disconnected. Let G be a graph with m edges, p cut sets. The cut-set matrix Q = [q_{ij}] be a p×m matrix of G. It is denoted by Q(G).where,

$$q_{ij} = \begin{cases} 1, if\ i^{th}\ cut - set\ contain\ j^{th}\ \ edge, \\ 0, otherwise \end{cases}$$

The cut set matrix typically has rows representing the cut sets and columns representing the edges of the graph. Each entry indicates whether an edge is present in the corresponding cut set.

Example

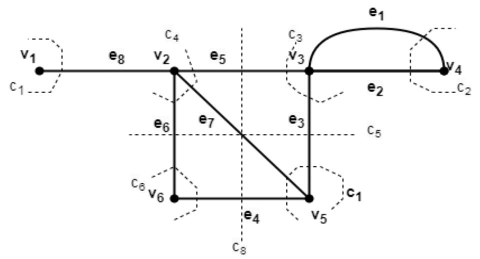


Figure 3.4.1. $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$

The cut-sets are $q_1 = \{e_8\}, q_2 = \{e_1, e_2\}, q_3 = \{e_3, e_5\}, q_4 = \{e_5, e_6, e_7\}, q_5 = \{e_3, e_6, e_7\},\ q_6 = \{e_4, e_6\}, q_7 = \{e_3, e_4, e_7\}\ and\ q_8 = \{e_4, e_5, e_7\},$

The cut set matrix of figure 3.2.6 is given by,

C(G) =

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
q_1	0	0	0	0	0	0	0	1
q_2	1	1	0	0	0	0	0	0
q_3	0	0	1	0	1	0	0	0
q_4	0	0	0	0	1	1	1	0
q_5	0	0	1	0	0	1	1	0
q_6	0	0	0	1	0	1	0	0
q_7	0	0	1	1	0	0	1	0
q_8	0	0	0	1	1	0	1	0

Applications of Cut-Set Matrix

The cut set matrix, also known as the incidence matrix for cuts, has applications in different fields, particularly in network analysis and system reliability. Some common applications include:

Telecommunication Network Design

Cut set matrices aid in designing and optimizing telecommunication networks by identifying critical links or nodes whose failure could disrupt communication.

Pipeline and Transportation Networks

In the design and maintenance of transportation and pipeline networks, cut set matrices help analyze the potential impact of failures on the overall system.

Path Matrix

Path is an open walk in which no vertex or edge can appear twice. Let G be a graph with m edges, and u and v be any two vertices in G. The path matrix for vertices u and v denoted by

P(u, v)=$[p_{ij}]$. qnumber of different paths between u and v and there are m edges, , it is q$\times m$ matrix and it is defined by ,

$$p_{ij} = \begin{cases} 1, if\ jth\ edge\ lies\ in\ the\ path \\ 0, otherwise. \end{cases}$$

Clearly, a path matrix is defined for a particular pair of vertices, the rows in P(u,v) correspond to different paths between u and v, and the columns corresponds to different edges in G.

Example

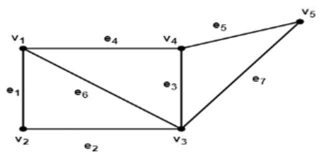


Figure 3.5.1

Consider the figure 3.2.7, with vertices $v_1, v_2, v_3, v_4\ and\ v_5$ there are three different paths from $v_1\ to\ v_3$.

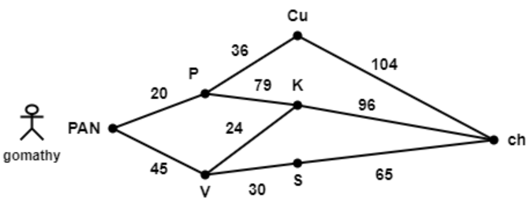
$P_1 = \{e_6\}, P_2 = \{e_4, e_3\}, P_3 = \{e_1, e_2\}$.

Seven edges in graph as $e_1, e_2, e_3, e_4, e_5, e_6\ and\ e_7$the path matrix $P(v_1, v_3)$ is of order 3$\times$ 7 is given the matrix ,

$$P(v_1, v_3) = \begin{matrix} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \\ p_1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ p_2 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ p_3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

Applications of Path Matrix

Geometry need to travel from panruti to Chidambaram. Remember that distances in this case refer to travel time in minutes. Find the shortest path to her travel?



PAN – panruti Ch - chidambaram

P - pattampakkam K - kallakuruchi S - sethiyathoppu

CU - cuddalore V - vadalore.

Solution:

Pan → Ch = Pan + P + CU + Ch = 160

Pan → Ch = Pan + P + K + Ch = 195

Pan → Ch = Pan + P + K + V + S +Ch = 218

Pan → Ch = Pan + P + V + S +Ch = 140

From this, we know that the shortest path from panruti to Chidambaram will take 140 minutes. Tracking which sequence of edges yielded 160 minutes, we see the shortest path is Pan → P →V →S →Ch

Proves and Problems

THEOREM

Let A = [a_{ij}] be adjacency matrix of a graph G with V = {$v_1, v_2, v_3, ..., v_p$}. Then (i,j)th entry of A^n is the number of walks of length n from v_i to v_j.

In otherwords, $[A^n]_{ij}$= Number of walks of length n from v_i to v_j.

Proof

We shall prove the theorem by induction on n.

Step 1

Prove that the theorem is true when n=1.

By the definition of adjacency matrix, a_{ij} = $\begin{cases} 1, if \text{ vi and vj } are\ adjacent, \\ 0, otherwise. \end{cases}$

Then a_{ij} = number of walks of length 1 from v_i to v_j .

⟹ (i,j)th entry of A^1 = number of walks of length from v_i to vj.

Hence the theorem is true when n = 1.

Step 2

Assume that the theorem is true when n = k.

Step 3

Prove that the theorem is true when n = k+1.

Consider, (i,j)th entry of (A^{k+1})

= (i,j)th entry of (A^k A)

=$\sum_{m=1}^{p}((i,m)^{th}\ entry\ of A^k)$ ((m,j)th entry of A)

=$\sum_{m=1}^{p}((i,m)^{th}\ entry\ of\ A^k)a_{mj}$

=$\sum_{m=1}^{p} number\ of\ walks\ of\ length\ k\ from\ v_i$ to $v_m)a_{mj}\ by\ step\ 2.$

By the definition of Adjacency matrix, $a_{mj} = \begin{cases} 1 \; if \; v_m \; and \; v_j \; are \; adjacent \\ \quad 0 \; otherwise \end{cases}$

So,

$(i,j)^{th}$ entry of A^{k+1} = $\sum_{m=1}^{p}(number \; of \; walks \; of \; length \; k + 1 \; from \; v_i \; to \; v_j \; via \; v_m$.

$\Longrightarrow (i,j)^{th}$ entry of A^{k+1} = number of walks of length k+1 from v_i to v_j.

$\Longrightarrow$ The theorem is true when n = k+1.

This proves the theorem.

Theorem

Ring sum of any 2 cut-sets in a graph is either a third cut-set or an edge disjoint union of cut – sets.

Proof

Let G be a graph and $s_1 \; and \; s_2$ are two cut – sets .

We know that , cut – set will partitioned the vertex of a graph.

So, $the \; cut - set \; s_1$ will partition the vertex set into two that is, $v_1 \; and \; v_2$.

The cut – set s_2 will partition the vertex set into two that is, $v_3 \; and \; v_4$.

$$v_1 \cup v_2 = v \;, v_1 \cap v_2 = \varphi$$

In such a way that , $v_3 \cup v_4 = v \,, v_3 \cap v_4 = \varphi$

Let , $v_5 = (v_1 \cap v_4) \cup (v_2 \cap v_3)$

$$v_6 = (v_1 \cap v_3) \cup (v_4 \cap v_2)$$

$v_5 = v_1 \oplus v_3$ and $v_6 = v_2 \oplus v_3$ ($\therefore$ $consider \; definition$ 3.1.14 $\oplus = ring \; sum$.)

Consider, $s_1 \oplus s_2$.

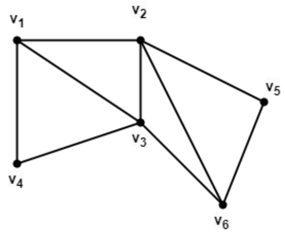
Edge belonging to this ring sum are only from $v_5 \; to \; v_6$ and also contains all the edges between $v_5 \; and \; v_6$.

Removal of $s_1 \oplus s_2$ disconnects G , partitions the vertex set into $v_5 \; and \; v_6$ such that ,

$$v_5 \cup v_6 = v \;, v_5 \cap v_6 = \varphi$$

This proves the theorem $s_1 \oplus s_2$is again a cut – set or an edge disjoint union of cut – sets.

How many walks are there from v_6 to v_4 of length 3?



Solution

The Adjacency Matrix of the Given Graph is,

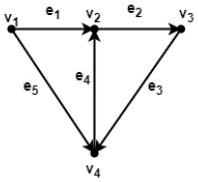
$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$$A.A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} . \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 2 & 1 & 1 & 2 \\ 1 & 4 & 2 & 2 & 1 & 2 \\ 2 & 2 & 4 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 & 0 & 1 \\ 1 & 1 & 2 & 0 & 2 & 1 \\ 2 & 2 & 1 & 1 & 1 & 3 \end{bmatrix}$$

$$A^2.A = A^3 = \begin{bmatrix} 3 & 1 & 2 & 1 & 1 & 2 \\ 1 & 4 & 2 & 2 & 1 & 2 \\ 2 & 2 & 4 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 & 0 & 1 \\ 1 & 1 & 2 & 0 & 2 & 1 \\ 2 & 2 & 1 & 1 & 1 & 3 \end{bmatrix} . \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 8 & 7 & 5 & 3 & 4 \\ 8 & 6 & 9 & 3 & 6 & 7 \\ 7 & 9 & 6 & 6 & 3 & 8 \\ 5 & 3 & 6 & 2 & 3 & 3 \\ 3 & 6 & 3 & 3 & 2 & 5 \\ 4 & 7 & 8 & 3 & 5 & 4 \end{bmatrix}$$

The walk from v_6 to v_4 of length 3 is 3.

Figure represents the graph of network. Find the incident matrix, reduced incident matrix and find the number of trees in the network.



Solution to find Incident matrix:

$$F(G)=(f_{ij}= \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{matrix} e_1 & e_2 & e_3 & e_4 & e_5 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & -1 \end{bmatrix} \end{array}$$

To find reduced incident matrix:

$$F(G) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \end{array} \begin{array}{c} \begin{matrix} e_1 & e_2 & e_3 & e_4 & e_5 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

To find the number of trees in the network:

$$F = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 \end{bmatrix}, \quad F^T = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$F.F^T=$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 \end{bmatrix} . \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

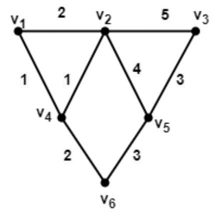
$$F.F^T = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$\det(F.F^T) = 2(6 - 1) + 1(-2)$

$= 10 - 2 = 8$

The number of trees in a graph is 8.

Observe the given weighted graph



Let the shortest distance between vertex v_1 and the vertex v_5 be d and the number of paths with distance d be p. What is the value of the expression $p^2 + 2pd + d^2$?

Solution

$v_1 \to v_2 \to v_5$ = 2 +4 = 6, $v_1 \to v_4 \to v_6 \to v_5$ = 1 + 2 + 3 = 6

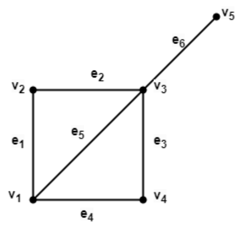
$v_1 \to v_4 \to v_2 \to v_5$ = 1 + 1 + 4 = 6

Distance = d = 6, Number of paths = p = 3

$p^2 + 2pd + d^2 = (p + d)^2 = (3 + 6)^2 = 9^2 = 81$

The value of $p^2 + 2pd + d^2$ *is* 81.

To find the ring sum of any 2 cut-sets in a graph is either a third cut-set or an edge disjoint union of cut – sets.



Solution

$s_1 = \{e_1, e_2\}$ can split into two partitions , $V_1 = \{v_2\}$, $V_2 = \{v_1, v_3, v_5 v_4\}$

$s_2 = \{e_1, e_5, e_4\}$ can split into two partitions , $V_3 = \{v_1\}$, $V_4 = \{v_3, v_2, v_4, v_5\}$

$v_5 = \cup (v_2 \cap v_3) = \{v_2\} \cup \{v_1\} = \{v_1, v_2\}$

$$v_6 = (v_1 \cap v_3) \cup (v_4 \cap v_2)$$

$$= \{\ \} \cup \{v_4, v_3, v_5\} = \{v_4, v_3, v_5\}$$

s_3 can split into $V_5 = \{v_1, v_2\}$, $V_6 = \{v_4, v_3, v_5\}$, we get ,

$$s_3 = \{e_2, e_5, e_4\}$$

$s_1 \oplus s_2 = \{e_2, e_5, e_4\}$

$$s_3 = s_1 \oplus s_2$$

∴Ring sum of any 2 cut-sets in a graph is again a cut-set.

Conclusion

In this work, graphs are discussed with simple examples and theorems to explain easily.

The historical background of graphs states that many important topics of science could not possible to explain without the help of graph theory. Many practical problems can be easily represented in terms of graph theory. The main aim of this paper is to present the importance of graph theoretical idea in matrix. In graph theory, matrices play a crucial role in representing and analyzing various properties of graphs.

These matrices serve as powerful tools for understanding graph properties, connectivity, shortest paths, and other fundamental aspects of graph theory.

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